Journal of Information Hiding and Multimedia Signal Processing ©2015 ISSN 2073-4212
Ubiquitous International Volume **6**, Number **4**, July **2015**

Fuzzy Adaptive Variational Bayesian Unscented Kalman Filter

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Received October, 2014; revised January, 2015

Abstract. *We consider the problem of nonlinear filtering under the circumstance of unknown covariance statistic of the measurement noise. A novel adaptive unscented Kalman filter (UKF) integrating variational Bayesian methods and fuzzy logic techniques is proposed in this paper. It is called fuzzy adaptive variational Bayesian UKF (FAVB-UKF). Firstly, the sufficient statistics of the measurement noise variances are estimated with a fixed-point iteration of the UKF in real time. Secondly, a fuzzy inference system (FIS) is introduced to adaptively adjust the measurement noise covariance based on a covariance matching technique. And last, the standard UKF with modified measurement noise covariance is carried out to obtain a state estimation. Simulation examples are used to evaluate the performance of this new algorithm comparing with UKF, and the results show that the proposed method is efficient and effective for potential practical applications*
Keywords: Nonlinear system; Adaptive filter; UKF; Variational Bayesian methods; Fuzzy inference system.

1. Introduction.

Nonlinear filtering problems abounds in many diverse fields including economics, statistics and numerous statistical and array processing engineering problems, such as time series analysis, communications, target tracking, and satellite navigation [1].

The Bayesian framework is the most commonly used method for the study of the nonlinear dynamic systems. And this most widely used filter in radar tracking application is extended Kalman filter (EKF), which is based upon the principles of linearizing the nonlinear models using Taylor series expansions [2]. Due to the assumptions of local linearization and the computation of the Jacobin matrix of the state vector, EKF may provide poor performance or divergences. To avoid this problem, the unscented Kalman filter (UKF) was proposed by Julier, et al. [3, 4]. Unlike the conventional EKF, where the linearization process is involved, the unscented Kalman filter employs a set of sigma points by deterministic sampling. When the sample points are propagated through the true nonlinear system, the posterior mean and covariance can be captured accurately to the 2nd order of Taylor series expansion for any nonlinear system. One of the remarkable merits is that the overall computational complexity of the UKF is the same as that of the EKF.

A serious limitation in the above methods is that they require a complete priori knowledge of the measurement and dynamic model parameters, including the noise statistics. The exact knowledge of the parameters, especially, the noise statistics, is not known in many practical situations a plausible assumption. The use of wrong prior statistics in UKF can lead to large estimation errors or even divergence. Variational Bayesian (VB) technology is a strong tool to treat state estimation with unknown noise variance [5, 6]. Recently, a high-efficiency variational Bayesian adaptive Kalman filter algorithm was presented [7]. which uses the VB method to approximate a joint posterior distribution of state and measurement noise.By a factorized free form distribution, it realizes suboptimal synchronous estimation of state and covariance of measurement noise. However, it is only available to linear systems. Afterward, by use of UKF and cubature Kalman filter(CKF), some variational Bayesian adaptive filters were proposed to cope nonlinear state estimation with unknown covariance of measurement noise in [8, 9]. In the research of artificial intelligence, the fuzzy theory has been extensively applied in describing some uncertain knowledge or physics phenomena [10, 11]. To deal with the noise uncertainty, the fuzzy inference system(FIS) is employed for dynamically on-line determining the measurement noise covariance according to the innovation information. Some fuzzy adaptive filters have been proposed [12, 13, 14]. Obviously, it is necessary to develop an adaptive nonlinear filter algorithm integrating VB methods and fuzzy theory, in order to enhance the estimation precision and fault-tolerance.

In this paper, VB and FIS are introduced to the traditional UKF, and an adaptive algorithm named fuzzy adaptive variational Bayesian UKF(FAVB-UKF) is proposed. The algorithm uses UKF to estimate the nonlinear system states, at the same time, it adaptively estimates and adjusts measurement noise variance through VB and FIS. The remainder of the paper is organized as follows. Section 2 presents the formulation of the problem. The variational Bayesian UKF(VB-UKF) is briefly reviewed in Section 3. FAVB-UKF is developed and briefly analyzed in Section 4 and Section 5, respectively. Simulation results that compare the performances of the existing algorithms are presented in Section 6. Finally, some conclusions are provided in Section 7.

2. **Problem formulation.** Consider a class of nonlinear dynamic system with additive noise, which is described as,

$$\mathbf{x}(k+1) = f(\mathbf{x}(k)) + \mathbf{w}(k) \tag{1}$$

$$\mathbf{z}(k) = h(\mathbf{x}(k)) + \mathbf{v}(k) \tag{2}$$

where $\mathbf{x}(k) \in \Re^{n\times 1}$ and $\mathbf{z}(k) \in \Re^{n\times 1}$ are the state of the system and the measurement at time step k, respectively; $f : \Re^n \to \Re^m$ is the nonlinear process model; $f : \Re^m \to \Re^n$ stands for the nonlinear observation model; $\mathbf{w}(k) \in \Re^{n\times 1}$ and $\mathbf{V}(k) \in \Re^{m\times 1}$ are the process noise and measurement noise with zero means and covariances $\mathbf{Q}(k)$ and $\mathbf{R}(k)$,respectively. It is supposed that the initial state $\mathbf{x}(0)$ and all noise vector are mutually statistically independent.

In some practical systems, such as target tracking system and the integrated navigation system, statistical properties of the measurement noise is very difficult to predict. The reasons are as follows: 1) It is hard to determine the exact values of the measurement noise covariance in advance. 2) There is a coupling error of coordinate transformation. 3) The dynamic system may be affected by various disturbance at any time, which results in changing the statistical properties of measurement noise.In a word, it is not reliable to describe measurement noise by using limited prior information. In this paper, the main problem is presented as follows: Aiming at the nonlinear system modelled by (1)and (2), when measurement noise statistics is unknown, how to design adaptive estimator.

3. **Variational Bayesian UKF.** In this section,the variational Bayesian UKF (VB-UKF) is briefly reviewed. In VB-UKF, Variational Bayesian (VB) technology is adopted to approximate a joint posterior distribution of state and measurement noise and realize suboptimal synchronous estimation of state and covariance of measurement noise by a factorized free form distribution. Formula of VB-UKF are summarized as follows [8]:

Algorithm1: Variational Bayesian UKF

Time update

1) Determine the sigma point and weighting coefficients

$$\begin{cases} \mathbf{X}_0(k-1|k-1) = \hat{\mathbf{x}}(k-1|k-1) \\ \mathbf{X}_j(k-1|k-1) = \hat{\mathbf{x}}(k-1|k-1) + (\sqrt{(n+\lambda)\mathbf{P}(k-1|k-1)})_j, j = 1,2,\cdots,n \\ \mathbf{X}_j(k-1|k-1) = \hat{\mathbf{x}}(k-1|k-1) - (\sqrt{(n+\lambda)\mathbf{P}(k-1|k-1)})_{j-n_x}, j = i+n,\cdots,2n \end{cases} \tag{3}$$

where n is the dimension of state $\mathbf{x}(k)$; $\hat{\mathbf{x}}(k-1|k-1)$ and $\mathbf{P}(k-1|k-1)$ are state estimate and its corresponding covariance at time $(k-1)$,respectively; $\lambda = a^2(n+k) - n$ is the scale parameter,α determines the degree of dispersion of sigma points, usually is set to a small positive number (e.g,0.01). κ is usually set to zero. $(\sqrt{(n+\lambda)\mathbf{P}(k-1|k-1)})_j$ meas the j-th row of matrix square root.

The weighting coefficients of mean and covariance are obtained as follows

$$\begin{cases} \omega_0^{(m)} = \lambda/(n+\lambda) \\ \omega_0^{(c)} = \lambda/(n+\lambda) + (1-\alpha^2+\beta) \\ \omega_j^{(m)} = \omega_j^{(c)} = 1/2(n+\lambda), j = 1,2,...,2n \end{cases} \tag{4}$$

where $\beta$is used to describe the distributed information(the optimal value is 2 under Gauss circumstance).

2) Compute the propagated sigma points

$$\mathbf{X}_j^*(k|k-1) = f(\mathbf{X}_j(k-1|k-1)) \tag{5}$$

3) Estimate the predicted state and its error covariance

$$\hat{\mathbf{x}}(k|k-1) = \sum_{j=0}^{2n} \omega_j^{(m)} \mathbf{X}_j^*(k|k-1) \tag{6}$$

$$\mathbf{P}(k|k-1) = \sum_{j=0}^{2n} \omega_j^{(c)} [\mathbf{X}_j^*(k|k-1) - \hat{\mathbf{x}}(k|k-1)][\mathbf{X}_j^*(k|k-1) - \hat{\mathbf{x}}(k|k-1)]^T + \mathbf{Q}(k-1) \tag{7}$$

4) Evaluate parameters predict of variance

$$\begin{cases} \zeta(k|k-1) = \rho \bullet \zeta(k-1) \\ \eta(k|k-1) = \rho \bullet \eta(k-1) \end{cases} \tag{8}$$

where"$\bullet$"is the point operation in Matlab. $\rho = [\rho_1,\cdots,\rho_m]^T$, $\rho_i \subset (0,1)(i = 1,\cdots,m)$,$\eta(k) = [\eta_1(k),\cdots,\eta_m(k)]^T$.

Measurement update (N iterations)

1) Determine the sigma points

$$\begin{cases} \mathbf{X}_0(k|k-1) = \hat{\mathbf{x}}(k|k-1) \\ \mathbf{X}_j(k|k-1) = \hat{\mathbf{x}}(k|k-1) + (\sqrt{(n+\lambda)\mathbf{P}(k|k-1)})_j, j = 1, 2, ..., n \\ \mathbf{X}_j(k|k-1) = \hat{\mathbf{x}}(k|k-1) - (\sqrt{(n+\lambda)\mathbf{P}(k|k-1)})_{j-n}, j = n+1, ..., 2n \end{cases} \tag{9}$$

2) Compute the propagated sigma points

$$\mathbf{Z}_j(k|k-1) = h(\mathbf{X}_j(k|k-1)) \tag{10}$$

3) Calculate the predicted measurement

$$\hat{\mathbf{z}}(k|k-1) = \sum_{j=0}^{2n} \omega_j^{(m)} \mathbf{Z}_j(k|k-1) \tag{11}$$

4)Evaluate the cross-covariance matrix

$$\mathbf{P}_{xz}(k|k-1) = \sum_{j=0}^{2n} \omega_j^{(c)} [\mathbf{X}_j(k|k-1) - \hat{\mathbf{x}}(k|k-1)][\mathbf{Z}_j(k|k-1) - \hat{\mathbf{z}}(k|k-1)]^T \tag{12}$$

5) Iteration initialization, namely $t = 1$ and give iteration number N(generally $N = 3$ and $1 \leq t \leq N$), and

$$\begin{cases} \zeta(k) = [1/2, 1/2, \cdots, 1/2]^T + \zeta(k|k-1) \\ \hat{\mathbf{x}}^1(k|k-1) = \hat{\mathbf{x}}(k|k-1) \end{cases} \tag{13}$$

6) Compute the covariance of measurement noise

$$\hat{\mathbf{R}}^t(k) = diag(\eta^t(k) \bullet / \zeta(k)) \tag{14}$$

where operation diag(*) means that diagonal entries of matrix * form a column vector.

7) Compute the innovation covariance matrix

$$\mathbf{P}_{zz}(k|k) = \sum_{j=0}^{2n} \omega_j^{(c)} [\mathbf{Z}_j(k|k-1) - \hat{\mathbf{z}}(k|k-1)][\mathbf{Z}_j(k|k-1) - \hat{\mathbf{z}}(k|k-1)]^T + \hat{\mathbf{R}}^t(k) \tag{15}$$

8) Calculate the Kalman gain

$$\mathbf{K}^t(k) = \mathbf{P}_{xz}(k|k)[\mathbf{P}_{zz}^t(k|k)]^{-1} \tag{16}$$

9) Estimate the updated state and its error covariance

$$\hat{\mathbf{x}}^t(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}^t(k)(\mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1)) \tag{17}$$

$$\mathbf{P}^t(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}^t(k)[\mathbf{P}_{zz}^t(k|k) + \hat{\mathbf{R}}^t(k)]^{-1}[\mathbf{K}^t(k)]^T \tag{18}$$

10) If $t < N$, update parameter $\eta^t(k)$

$$\eta^t(k) = \eta(k|k-1) + (\mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1)))^{\bullet 2}/2 + diag\{\mathbf{P}_{zz}(k|k)/2 \tag{19}$$

Then, let $t = t + 1$, and go back to step 1); else go to 11).

11) When $t = N$, the iteration finished ,and

$$\begin{cases} \eta(k) = \eta^N(k) \\ \hat{\mathbf{R}}(k) = \hat{\mathbf{R}}^N(k) \end{cases} \begin{cases} \hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}^N(k|k) \\ \mathbf{P}(k|k) = \mathbf{P}^N(k|k) \end{cases} \tag{20}$$

4. **Fuzzy Adaptive Variational Bayesian UKF.** In this section, an on-line adaptive scheme of the VB-UKF employing the principles of fuzzy logic is presented. The adaptation is in the sense of adaptively adjusting the measurement noise covariance matrix from data as they are obtained. In that sense, a fuzzy inference system (FIS) is used to obtain the adjusting factors for $\mathbf{R}_k = \hat{\mathbf{R}}(k)$. The basic idea behind the covariance-matching technique is to make the actual value of the covariance of the innovation sequence match its theoretical value [12]. A graphical representation of the Fuzzy adaptive VB-UKF (FAVB-UKF) is shown in Fig.1.

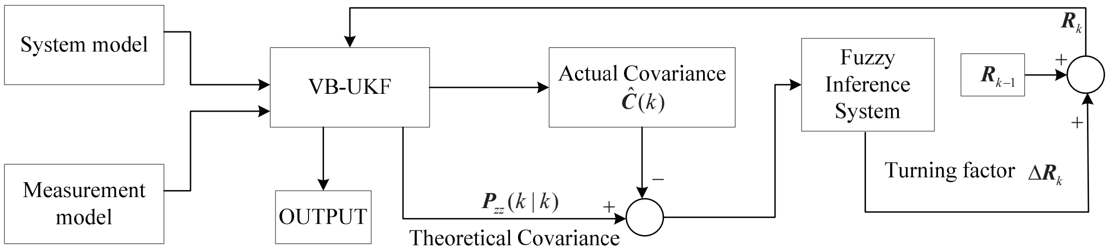

FIGURE 1. Membership functions for Structure of the FAVB-UKF

The innovation sequence$\gamma(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1)$ and its theoretical covariance $\mathbf{P}_{zz}(k|k) = \mathbf{P}_{zz}^N(k|k)$ obtained from the VB-UKF. Then, the actual covariance $\hat{\mathbf{C}}(k)$ is given by

$$\hat{\mathbf{C}}(k) = \frac{1}{W} \sum_{i=i_0}^{W} \gamma_i(k)\gamma_i^T(k) \tag{21}$$

where $i_0 = k - W + 1$ is the first sample inside the window. The window size W is chosen empirically to give some statistical smoothing.

Thus, if it is found that the actual covariance of $\gamma(k)$ has a discrepancy with its theoretical value, then adjustments have to be made to $\mathbf{R}_k$ in order to correct this mismatch. Now, a new variable called the Degree of Matching $\mathbf{DoM}_k$ is defined to indicate the degree of discrepancy between $\mathbf{P}_{zz}(k|k)$ and $\hat{\mathbf{C}}(k)$ as follows [8]:

$$\mathbf{DoM}_k = \mathbf{P}_{zz}(k|k) - \hat{\mathbf{C}}(k) \tag{22}$$

The main idea of adaptation used by a FIS is as follows. $\mathbf{R}_k$ can be used to vary $\mathbf{P}_{zz}(k|k)$ accordance with the value of $\mathbf{DoM}_k$ in order to reduce the discrepancies between $\mathbf{P}_{zz}(k|k)$ and $\hat{\mathbf{C}}(k)$. Because all matrices $\mathbf{R}_k$, $\mathbf{P}_{zz}(k|k)$, $\hat{\mathbf{C}}(k)$ and $\mathbf{DoM}_k$ are of the same dimension the adaptation of the (i,i) element of $\mathbf{R}_k$, can be made in accordance with the (i,i) element of $\mathbf{DoM}_k(i = 1, 2, \cdots, m)$. Thus,a single-input, $\mathbf{DoM}_k(i,i)$, single-output, $\Delta\mathbf{R}_k$, FIS can be used to sequentially generate the tuning or turning factors for the elements in the main diagonal of $\mathbf{R}_k$. The FIS is implemented considering three fuzzy sets for input $\mathbf{DoM}_k(i,i)$: N=Negative, ZE=Zero, and P=positive; and three fuzzy sets for output $\Delta\mathbf{R}_k$: I=Increase, M=Maintain, and D=Decrease; as in shown in Fig.2. The general rules of adaptation are as follows:

1) if $\mathbf{DoM}_k(i,i) = N$, then $\Delta\mathbf{R}_k = I$.
2) if $\mathbf{DoM}_k(i,i) = ZE$, then $\Delta\mathbf{R}_k = M$
3) if $\mathbf{DoM}_k(i,i) = P$, then $\Delta\mathbf{R}_k = D$

Then ,using the compositional rule of inference sum-prod and the center of area(COA) defuzzification method, the adjusting factor for the diagonal elements of $\mathbf{R}_k$, are sequentially obtained by a FIS, and the adjustments are performed in this way [15]

$$\mathbf{R}_k(i,i) = \mathbf{R}_{k-1}(i,i) + \Delta\mathbf{R}_k \tag{23}$$

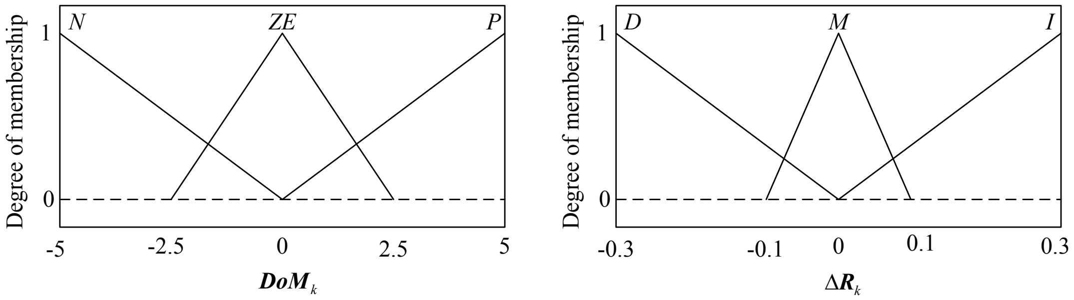

FIGURE 2. Membership functions for $\mathbf{DoM}_k$ and $\Delta\mathbf{R}_k$

In view of the above analysis, running formula of the FAVB-UKF are summarized as follows:

Algorithm2: Fuzzy Adaptive Variational Bayesian UKF

Step1: Parameter Initialization

Set initial state $\mathbf{x(0)}$, covariance $\mathbf{x(0)}$ and the size of moving estimation window W.

Step2: VB-UKF is carried out inside a moving estimation window

VB-UKF is used to calculate the measurements predictions $\hat{\mathbf{z}}_\mathbf{i}(k|k-1)$,$(i = i_0, \cdots, k)$, and covariance of innovations $\mathbf{P}_{zz}(k|k)$ by equations (3)-(20).

Step3: Adjust measurement noise covariance with FIS

Firstly, actual covariance $\hat{\mathbf{C}}(k)$ and Degree of Matching $\mathbf{DoM}_k$ are computed by equation(21) and (22), respectively. And then, FIS is used to modify $\mathbf{R}_k$ by equation (23).

Step4: Update state and covariance

In this step, standard UKF with adjusted measurement noise covariance $\mathbf{R}_k$ is adopted to update state and the covariance by equations (3)-(12), and (15)-(18).

5. **Brief Analysis.** In this section, brief analysis of novel algorithm is presented. As described previously, the standard UKF formulation assumes complete a priori knowledge of the noise statistics. However, in most practical applications measurement noise statistics are initially estimated or, in fact, unknown. The problem here is that the performance of UKF is closely connected to the quality of these priori noise statistics. Evidences have shown how a poor estimation of the input noise statistics may seriously degrade the filter performance, and even provoke the divergence of the filter. In view of this, a new adaptive UKF based on variational Bayesian methods and fuzzy logic techniques is developed, which is called fuzzy adaptive variational Bayesian UKF (FAVB-UKF). Compared with standard UKF, the advantages of FAVB-UKF are as follows:

(1) FAVB-UKF relaxes the assumptions of a priori knowledge of the noise statistics.

(2) For the existing fuzzy adaptive filters, initial measurement noise covariance is indispensable. However, in FAVB-UKF, it is not necessary. This is because the estimation of noise covariance by VB method can be considered as a initial value.

(3) A fuzzy inference system (FIS) is introduced to adaptively adjust the measurement noise covariance based on a covariance matching technique, which can avoid filter divergence and improve the robustness.

6. **Numerical Example.** Simulation experiment have been carried out to evaluate the performance of the proposed approach (FAVB-UKF) in comparison with the conventional UKF and variational Bayesian UKF(VB-UKF)[8]for target tracking scenario.

The targets are modeled as constant velocity objects in a plane with process noise (Gaussian zero mean) that accounts for slight changes in the velocities. More specifically, let a state vector $\mathbf{x}(k) = [x(k)\ \dot{x}(k)\ y(k)\ \dot{y}(k)]^{\mathrm{T}}$ where $x(k)$ are the position components of east and north,respectively.$\dot{x}(k)$and$\dot{y}(k)$ are the corresponding velocity components respectively. the state model is given by

$$f(\mathbf{x}(k)) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \mathbf{x}(k) + \mathbf{w}(k)$$

where sampling interval T=1s. The parameters of the target is given by initial state: $[1500\text{m}, 150\text{m/s},\ 1200\text{m}, 120\text{m/s}]^T$,$\mathbf{P}_0 = \mathrm{d}iag\{(50\text{m})^2, (10\text{m/s})^2, (50\text{m})^2, (10\text{m/s})^2\}$. $\mathbf{w}(k)$ is zero mean Gaussian noises with known covariance matrix $\mathbf{Q}(k)$, which is given by

$$\mathbf{Q}(k) = 0.25 \times diag(\mathbf{Q}1, \mathbf{Q}1), \text{where} \quad \mathbf{Q}1 = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix}$$

Radar is fixed at the origin of the plane and equipped to measure the range. Hence, the measurement equation is written as:

$$\mathbf{z}(k) = \sqrt{x^2(k) + y^2(k)} + \mathbf{v}(k)$$

where $\mathbf{v}(k)$is an additive zero-mean Gaussian noise vector with variance $\mathbf{R}(k)$.In this simulation experiment,$\mathbf{R}(k)$is designed to change according to

$$\mathbf{R}(k) = \begin{cases} \mathbf{R}_0 & 0 \le k \le 10 \\ 10\mathbf{R}_0 & 11 \le k \le 30 \\ 2\mathbf{R}_0 & \text{other} \end{cases}, \text{where} \quad \mathbf{R}_0 = \hat{\mathbf{R}}(0) = 4\text{m}^2$$

The parameters of UKF are chosen as :$\alpha = 0.01$, $\kappa = 0$, $\beta = 2$. The parameters of VB-UKF are set as $\rho = 0.001$, $\zeta(k) = 1$, $\eta(k) = 0.1$, $N = 3$.

For performance comparison, we compute the accumulative absolute error (AAE) in position and velocity. and mean AAE (MAAE). We define the AAE and MAAE in position

$$\text{AAE}_{\text{pos}}(k) = |x(k) - \hat{x}(k)| + |y(k) - \hat{y}(k)|; \quad \text{MAAE}_{\text{pos}}(k) = \frac{1}{N_s}\sum_{k=1}^{N_s} \text{AAE}_{\text{pos}}(k)$$

where $(x(k), \hat{x}(k))$ and $(y(k), \hat{y}(k))$ are the true and estimated positions at time k. N_s is the simulation steps. Similarly to the AAE_{pos}(k) and MAAE_{pos} ,we may also write formula of the AAE_{vel}(k) and MAAE_{vel}. Simulation steps $N_s = 60$ in this example.

Example 1:UKF VS FAVB-UKF In this example, tracking performance of UKF and FAVB-UKF is demonstrated and the simulation results are shown by Fig. 3, Fig. 4, and Table 1.

TABLE 1. The mean accumulative absolute error of two algorithms

MAAE	Algorithm	
	FAVB-UKF	UKF
position(m)	12.3564	98.7973
Velocity(m/s)	1.9398	4.1106

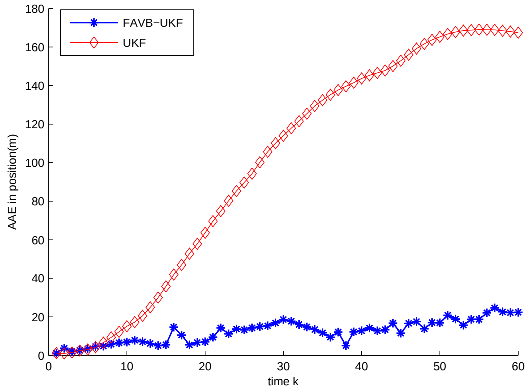

FIGURE 3. The accumulative absolute error in position

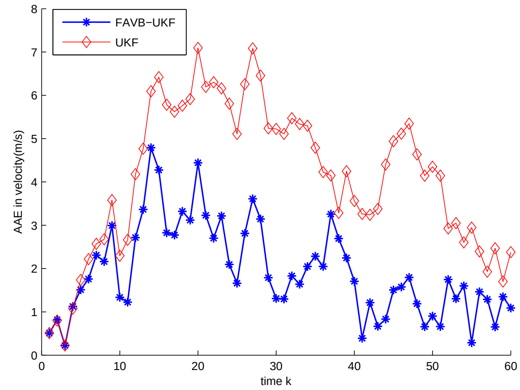

FIGURE 4. The accumulative absolute error in velocity

Fig. 3 and Fig.4 show the AAE for the two methods. From two figures, it can be seen that FAVB-UKF performs significantly than UKF. Compared with FAVB-UKF, the state estimate of UKF deviated from the true state very large. Note that as expected in Table1, the mean accumulative absolute errors of FAVB-UKF are obviously less than that of standard UKF. The reason is that the FAVB-UKF can estimate the unknown measurement noises online whereas the standard UKF depends on the fixed prior knowledge about the measurement noises.

Example 2:VB-UKF VS FAVB-UKF In this simulation, two adaptive algorithms are compared.The results are given from Figure 5 to Figure 7, and Table 2.

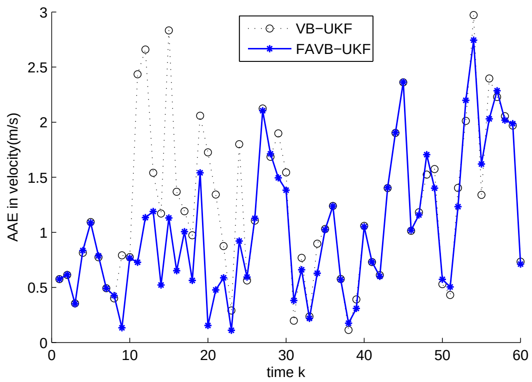

FIGURE 5. The accumulative absolute error in position

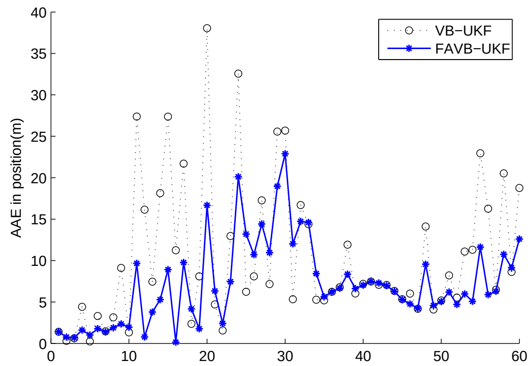

FIGURE 6. The accumulative absolute error in velocity

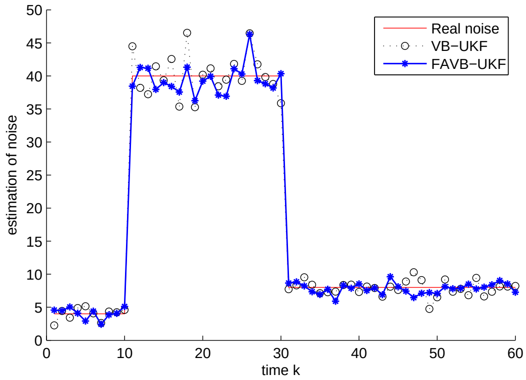

FIGURE 7. Estimation of measurement noise variance

TABLE 2. The mean accumulative absolute error of two algorithms

MAE	Algorithm	
	FAVB-UKF	VB-UKF
$\hat{\mathbf{R}}(k)$	1.0010	1.3556

From Fig.5 and Fig.6, it is easy to see that FAVB-UKF obtains better estimation accuracy than VB-UKF. In Table 2, the MAAE of two methods also proved this. Obviously, FUKF not only can estimate the variance of measurement noise similar to VB-UKF, but also be able to adjust the estimation results. Estimation error of the noise variance also confirmed this argument in Fig.7 and Table2 . These results are consistent with our analysis in section 5. It means that the proposed method is an effective approach for real application.

7. **Conclusions.** In order to deal with the noise uncertainty and system nonlinearity simultaneously, VB technology and FIS are introduced in traditional UKF, a new adaptive UKF is presented in this article. The simulation results show that the proposed method performs excellently, and it is an efficient algorithm for the real application. In the future, we would like to consider the system with unknown process noise and measurement noise. On the basis of these adaptive methods, it will also be interesting to design nonlinear fusion algorithms in centralized and distributed fusion framework. Furthermore, another interesting future research topic is to compare the computational demands of various adaptive filters.

Acknowledgment. This work was partially supported by Zhejiang Provincial Natural Science Foundation of China under Grant No.LQ14F030001, and also supported by the National Natural Science Foundation of China under Grant No.61403218. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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