Journal of Information Hiding and Multimedia Signal Processing ©2017 ISSN 2073-4212
Ubiquitous International Volume **8**, Number **3**, May **2017**

DOA Estimation Method for Two-Dimensional Wideband Signals by Sparse Recovery in Frequency Domain

Jia-Qi Zhen, Zhi-Fang Wang

Department of Electronic Engineering
Heilongjiang University
No.74, Xuefu Road, Nangang District, Harbin, China
zhenjiaqi2011@163.com

Received September, 2016; revised February, 2017

Abstract. *The direction of arrival(DOA) estimation methods with sparse recovery need to separate the searching space into some angle meshes, which will cause some error to the result. Therefore, a novel approach for two-dimensional wideband signals by sparse recovery is used, the iterative functions are constructed and calculated by output signal, after that the support set is acquired according to the root finding and corresponding semi-definite programming, then the DOA is acquired by integrating the data of all frequency bins, meanwhile the signal can also be recovered in continuous frequency domain. It avoids the error brought by sparse recovery in discrete domain and particularly performs well when the number of snapshots is small.*
Keywords: Direction of arrival (DOA) estimation, Sparse recovery, Wideband signals, Frequency domain

1. **Introduction.** Direction of arrival(DOA) estimation is widely used in source localization [1-6], radio monitoring[7-9] and electronic countermeasure[10-11], one of the techniques is based on sparse recovery, it is a novel kind of super-resolution direction finding methods arising in recent years, but most of them are only used for narrowband signals. The joint sparse property of all frequency bins is a remarkable advantage for wideband array signal processing based on fractional frequency[12]. Hyder[13] calculated DOA of near wideband and narrowband signals combing norm when the sampling is small. Liu[14] used the homotopy for basis pursuit, obtained the optimal adjust parameters and recovery result by iteration. Liu[15] solved the wideband DOA under the condition of sparse constraint, and reconstruct the signal by the spatial sparse property of multiple frequency bins, this was a expansion of the standard sparse Bayesian learning algorithm. Eldar[16] proposed the idea of block sparse, then Hu[17] calculated DOA of wideband signal based on block sparse recovery, solved the complicated calculation of traditional sparse recovery.

The sparse recovery proposed in recent years have lowered the demand to number of samplings relative to the traditional focusing methods, and increased the estimation accuracy, but they are usually confined by off-line effect. So Tan[18] presented an iterative grid optimization algorithm, determined the original signals and mesh mismatches synchronously. Candes[19-20] studied the signal recovery with continuous model, avoided off-line effect when signals were reconstructed with discrete model, then the estimation had been improved to a large extent, but he did not tell us the detail process of wideband DOA estimation.

In this paper, a novel two-dimensional DOA estimation method is proposed, first, the wideband signals are separated into some non-overlapped frequency bins, the iterative functions are constructed and calculated by output signal, after that the support set is acquired according to the root finding and corresponding semidefinite programming, then the DOA is acquired by integrating the data of all frequency bins, meanwhile the signal can also be recovered in continuous frequency domain. This method avoids the error brought by sparse recovery in discrete domain, it particularly performs well when the number of snapshots is small.

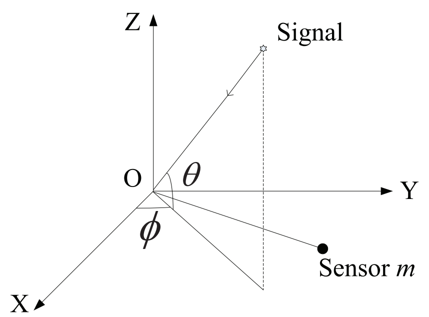

FIGURE 1. Array signal model

2. **Array signal model.** It is seen from FIGURE 1, consider an arbitrary array with M sensors in X-Y plane, the phase reference of the array is defined as the origin, and the coordinate of the mth sensor is (x_m, y_m) $(m = 1, 2, \cdots M)$. Assume that there are N far-field wideband signals arriving at the array, angles of arrival are (ϕ_n, θ_n) $(n = 1, 2, \cdots N)$, where ϕ_n and θ_n are respectively defined as the azimuth and elevation, so output of M sensors can be written:

$$\begin{aligned}\boldsymbol{y}(t) &= [y_1(t), \cdots, y_m(t), \cdots, y_M(t)]^{\mathrm{T}} = [\sum_{n=1}^{N} s_n(t-\tau_{1n}), \cdots, \sum_{n=1}^{N} s_n(t-\tau_{mn}), \cdots, \\ &\sum_{n=1}^{N} s_n(t-\tau_{Mn})]^{\mathrm{T}} + [b_1(t), \cdots, b_m(t), \cdots, b_M(t)]^{\mathrm{T}} \end{aligned} \tag{1}$$

where $\tau_{mn} = \frac{x_m cos\phi_n cos\theta_n + y_m sin\phi_n cos\theta_n}{c}$ is the propagation delay for the nth signal arriving at the mth sensor with respect to the reference of the array, $y_m(t)$ is the output of the mth sensor, $b_m(t)$ is the corresponding additive Gaussian white noise, c is the propagating speed of the signal.

Suppose number of the subband is K, the array output can be written as

$$\boldsymbol{Y}(f_k) = \boldsymbol{A}(f_k)\boldsymbol{S}(f_k) + \boldsymbol{B}(f_k) \ (k = 1, 2, \cdots K) \tag{2}$$

the steering vector matrix at frequency f_k is

$$\begin{aligned}\boldsymbol{A}(f_k) &= [\boldsymbol{a}(f_k,\phi_1,\theta_1),\boldsymbol{a}(f_k,\phi_n,\theta_n),\boldsymbol{a}(f_k,\phi_N,\theta_N)] \\ &= \begin{bmatrix} e^{-j2\pi f_k\tau_{11}}\dots\ e^{-j2\pi f_k\tau_{1n}}\dots\ e^{-j2\pi f_k\tau_{1N}} \\ \vdots \\ e^{-j2\pi f_k\tau_{m1}}\dots\ e^{-j2\pi f_k\tau_{mn}}\dots\ e^{-j2\pi f_k\tau_{mN}} \\ \vdots \\ e^{-j2\pi f_k\tau_{M1}}\dots\ e^{-j2\pi f_k\tau_{Mn}}\dots\ e^{-j2\pi f_k\tau_{MN}} \end{bmatrix}\end{aligned} \tag{3}$$

here, $\boldsymbol{a}(f_k,\phi_n,\theta_n)$ is the direction vector of wideband signal coming from $(\phi_n,\theta_n)(n=1,2,\cdots N)$ at frequency f_k, suppose $\boldsymbol{S}(f_k)$ is composed by several spikes[21-22], define

$$\varphi_n(f_k)=\frac{f_k}{c}[1-(cos\phi_n cos\theta_n+sin\phi_n cos\theta_n)] \tag{4}$$

so $\boldsymbol{S}(f_k)$ can be expressed as the sparse form below:

$$\boldsymbol{S}_k(f_k)=\begin{bmatrix} S_1(f_k) \\ \vdots \\ S_n(f_k) \\ \vdots \\ S_N(f_k)\end{bmatrix}=\begin{bmatrix} \upsilon_1{}^2(f_k)\delta_{\varphi_1(f_k)} \\ \vdots \\ \upsilon_n{}^2(f_k)\delta_{\varphi_n(f_k)} \\ \vdots \\ \upsilon_N{}^2(f_k)\delta_{\varphi_N(f_k)}\end{bmatrix} \tag{5}$$

and $\delta_{\varphi_n(f_k)}$ is a dirac measure located at $\varphi_n(f_k)$, define $\varphi_1(f_k),\cdots\varphi_N(f_k)$ as the sparse support set of the signals, $\upsilon_n{}^2(f_k)$ is defined as its power.

3. Principle of the method.

3.1. Infinite samples. Suppose the received data $\boldsymbol{Y}(f_k)$ is infinite, it is expressed by (2), given a measure $S(\varphi)$, the Fourier coefficients can be expressed as

$$q(m)=\int_0^1 exp(-j2\pi m\varphi)S(\varphi)d\varphi\ (m=1,2,\cdots M) \tag{6}$$

combine (5) and (6), we have

$$q(m,f_k)=\sum_{n=1}^{N} exp(-j2\pi m\varphi_n(f_k))\upsilon_n{}^2(f_k)\ (m=1,2,\cdots M) \tag{7}$$

thus, equation(7) can simply be expressed as

$$\boldsymbol{Q}(f_k)=\boldsymbol{F}(f_k)\boldsymbol{S}(f_k) \tag{8}$$

where

$$\boldsymbol{Q}(f_k)=[q(1,f_k),q(2,f_k),\cdots q(M,f_k)]^{\mathrm{T}} \tag{9}$$

and

$$\boldsymbol{F}(f_k)=\begin{bmatrix} exp(-j2\pi\varphi_1(f_k)\cdots\ exp(-j2\pi\varphi_N(f_k)) \\ exp(-j2\pi\times2\varphi_1(f_k)\cdots\ exp(-j2\pi\times2\varphi_N(f_k)) \\ \vdots \\ exp(-j2\pi M\varphi_1(f_k)\cdots\ exp(-j2\pi M\varphi_N(f_k))\end{bmatrix} \tag{10}$$

in order to reconstruct initial wideband signal, it is possible to solve the following question

$$\min_{\boldsymbol{S}(f_k)}\sum_{k=1}^{K}\|\boldsymbol{S}(f_k)\|_{\mathrm{TV}},\ \text{s.t.}\boldsymbol{Q}(f_k)=\boldsymbol{F}(f_k)\boldsymbol{S}(f_k),(k=1,2,\cdots K) \tag{11}$$

where $\|\boldsymbol{S}(f_k)\|_{\mathrm{TV}} = \sum_{n=1}^{N} S_n(f_k) = \sum_{n=1}^{N} \upsilon_n{}^2(f_k)$, it is the total-variation norm of a measure, then we can recover the signal $\boldsymbol{S}(f_k)$ if the distance between $\varphi_\alpha(f_k)$ and $\varphi_\beta(f_k)$ is wider than $2/f_k$ for $1\leq\alpha,\beta\leq N$, $\alpha\neq\beta$; $k = 1, 2, \cdots K$[19];

3.2. **Finite samples.** In fact, the received data $\boldsymbol{Y}(f_k)$ is not always infinite, thus $\boldsymbol{Y}(f_k)$ is composed of finite samples, assume that the number of snapshots at every frequency is KP, equation(2) needs to be modified as

$$\overline{\boldsymbol{Y}}(f_k) = \boldsymbol{A}(f_k)\overline{\boldsymbol{S}}(f_k) + \overline{\boldsymbol{B}}(f_k), (k = 1, 2, \cdots K) \tag{12}$$

where

$$\overline{\boldsymbol{Y}}(f_k) = [\boldsymbol{Y}(f_k, 1), \cdots, \boldsymbol{Y}(f_k, kp), \cdots, \boldsymbol{Y}(f_k, KP)] \tag{13}$$

here $\boldsymbol{Y}(f_k, kp)$ is the kpth sampling vector of the received data at f_k, $\overline{\boldsymbol{S}}(f_k)$ and $\overline{\boldsymbol{B}}(f_k)$ are respectively the finite signal and noise sampling vector. Remove the noise item from both sides of (12), we have

$$\overline{\boldsymbol{Y}}(f_k) - \overline{\boldsymbol{B}}(f_k) = \boldsymbol{A}(f_k)\overline{\boldsymbol{S}}(f_k) = \boldsymbol{A}(f_k)\boldsymbol{S}(f_k) + \boldsymbol{D}(f_k) \tag{14}$$

here $\boldsymbol{D}(f_k)$ can be deemed as a perturbing term, it reflects the error between infinite and finite received data. Combining (14), we can deduce the Fourier coefficients of finite samples

$$\begin{aligned}
q(m, f_k) &= exp(-j2\pi m\frac{f_k}{c})(\overline{\boldsymbol{Y}}_m(f_k) - \overline{\boldsymbol{B}}_m(f_k)) \\
&= exp(-j2\pi m\frac{f_k}{c})(\sum_{n=1}^{N} exp(j2\pi m\frac{f_k}{c}(cos\phi_n cos\theta_n + sin\phi_n cos\theta_n)\upsilon_n{}^2(f_k) + \\
&\quad \boldsymbol{D}(m, f_k))) \\
&= \sum_{n=1}^{N} exp(-j2\pi m\frac{f_k}{c}(1 - (cos\phi_n cos\theta_n + sin\phi_n cos\theta_n)))\upsilon_n{}^2(f_k) + \\
&\quad exp(-j2\pi m\frac{f_k}{c})\boldsymbol{D}(m, f_k) \\
&= exp(-j2\pi m\varphi_n(f_k))\upsilon_n{}^2(f_k) + \boldsymbol{\omega}(m, f_k)
\end{aligned} \tag{15}$$

where $\boldsymbol{\omega}(m, f_k) = exp(-j2\pi m\frac{f_k}{c})\boldsymbol{D}(m, f_k)$, so (15) needs to be modified as

$$\boldsymbol{Q}(f_k) = \boldsymbol{F}(f_k)\boldsymbol{S}(f_k) + \boldsymbol{\omega}(f_k) \tag{16}$$

where $\boldsymbol{\omega}(f_k) = [\boldsymbol{\omega}(1, f_k), \cdots, \boldsymbol{\omega}(1, f_k)]^{\mathrm{T}}$, similarly, initial signal can be recovered with the data of all frequency bins by the following problem

$$\min_{\boldsymbol{S}(f_k)} \sum_{k=1}^{K} \|\boldsymbol{S}(f_k)\|_{\mathrm{TV}}, \text{ s.t.} \sum_{k=1}^{K} \|\boldsymbol{Q}(f_k) - \boldsymbol{F}(f_k)\boldsymbol{S}(f_k)\|_2 \leq \sum_{k=1}^{K} |\varsigma(f_k)| \tag{17}$$

the problem (17) above is a multiple convex program, they are complex to be calculated directly, we can solve the following dual problem of (17) to simplify them[20]

$$\begin{aligned}
&\max_{\boldsymbol{\phi}(f_k), \boldsymbol{Z}} \sum_{k=1}^{K} (\mathrm{Re}[\boldsymbol{Q}^*(f_k)\boldsymbol{\phi}(f_k)] - \varsigma(f_k)\|\boldsymbol{\phi}(f_k)\|_2) \text{ s.t.} \\
&\qquad \begin{bmatrix} \boldsymbol{Z} \ \boldsymbol{\phi}(f_k) \\ \boldsymbol{\phi}^*(f_k) \ 1 \end{bmatrix} \succ 0, k = 1, 2, \cdots, K, \\
&\qquad \sum_{k=1}^{K} \|\boldsymbol{F}^*(f_k)\boldsymbol{\phi}(f_k)\|_{L_\infty} \leq K
\end{aligned} \tag{18}$$

where

$$\sum_{\alpha=1}^{M-\beta} \boldsymbol{Z}_{\alpha,\alpha+\beta} = \begin{cases} 1, & \beta = 0 \\ 0, & \beta = 1, 2, \cdots, M-1 \end{cases} \tag{19}$$

$\boldsymbol{Z} \in C^{M \times M}$ is a Hermitian matrix,and $\boldsymbol{\phi}(f_k)$ is the corresponding Lagrangian multiplier matrix for $\boldsymbol{Q}(f_k) = \boldsymbol{F}(f_k)\boldsymbol{S}(f_k) + \boldsymbol{\omega}(f_k)$, it can be acquired via semidefinite program [23-24], which can be solved by SeDuMi[25] and CVX[26].

We can use the lemma below to indicate the relation between the solution of (17) and (18):

$$(\hat{\boldsymbol{F}}^* \hat{\boldsymbol{\phi}})(f_k) = \text{sign}(\|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}}), k = 1, 2, \cdots, K \tag{20}$$

where $\|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}} \neq 0$, $\hat{\boldsymbol{F}}(f_k)$,$\hat{\boldsymbol{\phi}}(f_k)$ and $\hat{\boldsymbol{S}}(f_k)$ are respectively the estimated vector of $\boldsymbol{F}(f_k)$,$\boldsymbol{\phi}(f_k)$ and $\boldsymbol{S}(f_k)$.

Proof: Combining the Cauchy Schwarz inequality, there is

$$\begin{aligned} \|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}} &= \sum_{n=1}^{N} S_n(f_k) = \text{Re} < \boldsymbol{Q}^*(f_k), \hat{\boldsymbol{\phi}}(f_k) > -\varsigma(f_k)\|\hat{\boldsymbol{\phi}}(f_k)\|_2 \\ &= < \hat{\boldsymbol{F}}(f_k)\hat{\boldsymbol{S}}(f_k), \hat{\boldsymbol{\phi}}(f_k) > + < \hat{\boldsymbol{Q}}(f_k) - \hat{\boldsymbol{F}}(f_k)\hat{\boldsymbol{S}}(f_k), \hat{\boldsymbol{\phi}}(f_k) > -\varsigma(f_k)\|\hat{\boldsymbol{\phi}}(f_k)\|_2 \\ &\leq < \hat{\boldsymbol{S}}(f_k), \boldsymbol{F}^*(f_k)\hat{\boldsymbol{\phi}}(f_k) > \end{aligned} \tag{21}$$

By Holders equality and $\|\hat{\boldsymbol{F}}^*(f_k)\hat{\boldsymbol{\phi}}(f_k)\|_{L_\infty} \leq 1$, we have $\|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}} \geq < \hat{\boldsymbol{S}}(f_k), \hat{\boldsymbol{F}}^*(f_k)\hat{\boldsymbol{\phi}}(f_k) >$, then $\|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}} =< \hat{\boldsymbol{S}}(f_k), \hat{\boldsymbol{F}}^*(f_k)\hat{\boldsymbol{\phi}}(f_k) >$, which holds (20).

As $\|\hat{\boldsymbol{S}}(f_k)\|_{\text{TV}} \neq 0$, take absolute value of both sides of (20), we have

$$|\hat{\boldsymbol{F}}^*(f_k)\hat{\boldsymbol{\phi}}(f_k)| = 1 \tag{22}$$

it can be seen from (22), if the information of $\boldsymbol{F}^*(f_k)\boldsymbol{\phi}(f_k)$ corresponds to the signal, it has modulus equal to one, otherwise, we have[19]

$$|\boldsymbol{F}^*(f_k)\boldsymbol{\phi}(f_k)| < 1 \tag{23}$$

so using the data of all the frequency bins, we have

$$\sum_{k=1}^{K} |\hat{\boldsymbol{F}}^*(f_k)\hat{\boldsymbol{\phi}}(f_k)| = K \tag{24}$$

then combining (4) and (10), we can estimate the DOA, meanwhile, the signal vector in continuous frequency domain can be recovered by (5) if we do not consider their powers.

As the proposed two-dimensional sparse recovery method is accomplished in continuous frequency domain, it can be called TSRF method for short.

4. **Simulations.** Here, we present some simulations with matlab below, consider several wideband chirp signals impinging on a plane array with 8 omnidirectional sensors, the center frequency of the signals is 4GHz, the coordinates are randomly given as (0,0), (-0.16,0.12), (-0.049, 0.086), (-0.22,0.055), (-0.079,-0.032), (0.065, 0.13), (0.08,0.24), (0.037,-0.044), unit is meter. two-sided correlation transformation(TCT) method[27], sparse recovery methods in discrete domain (SRD) and in frequency domain (TSRF) are respectively used for the simulations, $\varsigma(f_k)(k = 1, 2, \cdots, K)$ in TSRF method is taken as 1. The meshes of SRD are separated to (ϕ, θ) , the area of them are both $(0° \sim 90°)$, step size of SRD and searching step size of TCT are both $0.4°$, homotopy approach[29] is used for basis pursuit in SRD, 200 Monte-Carlo simulations have run for each condition.

4.1. **Normalization Spectrum.** Suppose three far-field wideband signals are respectively arriving at the array with same power from $(35.5°, 63.5°)$,$(48.5°, 50.5°)$, $(60.5°, 40°)$, SNR is 1dB, number of sampling at each frequency is 25, the width of the band is 25% of the center frequency, normalization spectrums are shown in FIGURE 2, 3 and 4. We

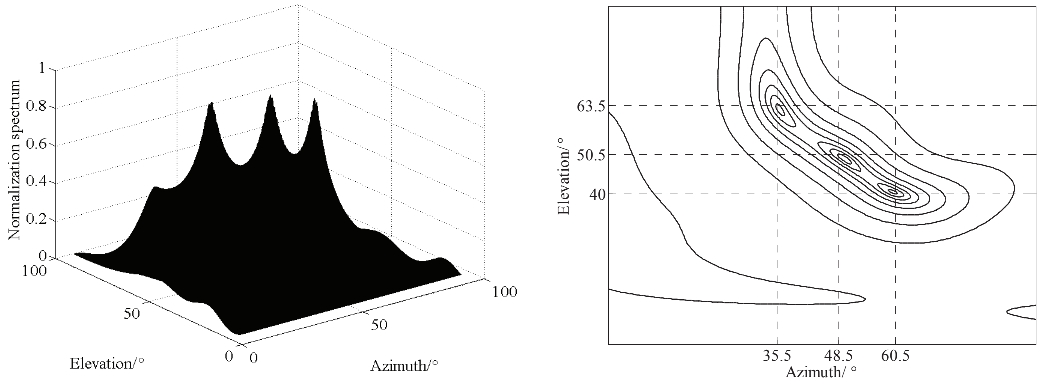

FIGURE 2. Normalization spectrum of TCT

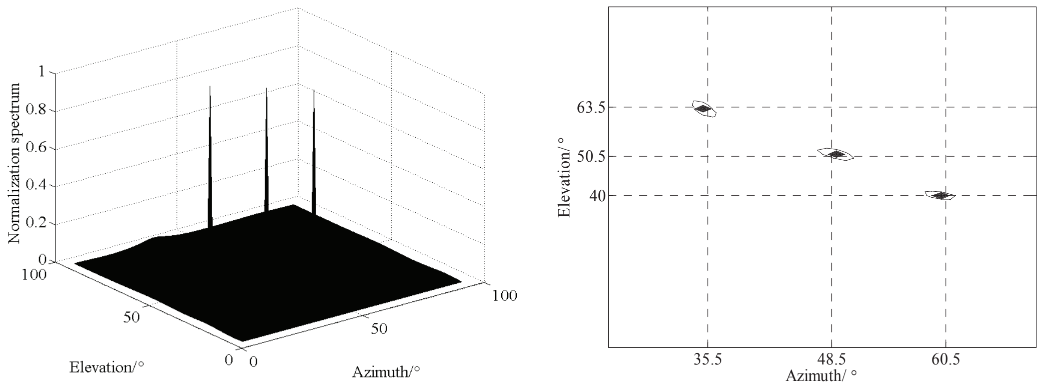

FIGURE 3. Normalization spectrum of SRD

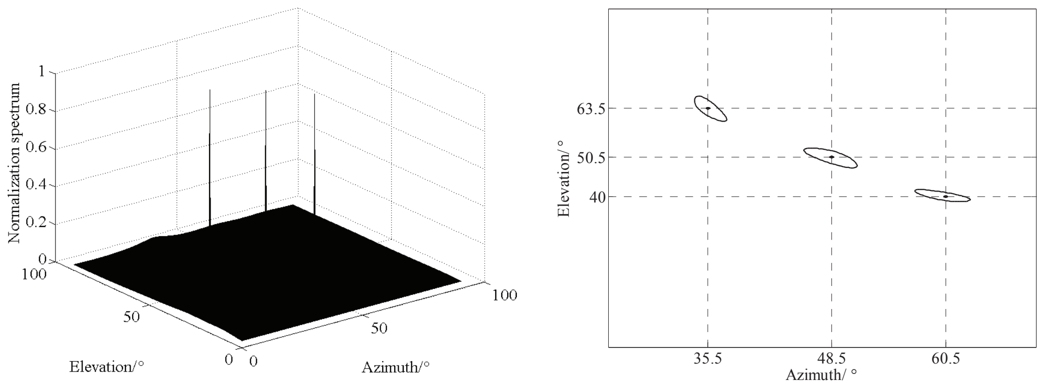

FIGURE 4. Normalization spectrum of TSRF

can see from FIGURE 2∼4, the normalization spectrum of TSRF is sharper than that of SRD and TCT. Due to the off-line effect, there is certain error for SRD, and we can not avoid it even if we use a more careful division.

4.2. **Estimation Error versus SNR.** Assume that SNR varies from -10dB to 10dB, then the estimation error versus SNR is shown in FIGURE 5 when number of snapshots at each frequency is 20. It can be seen from FIGURE 5, TSRF method can estimate DOA

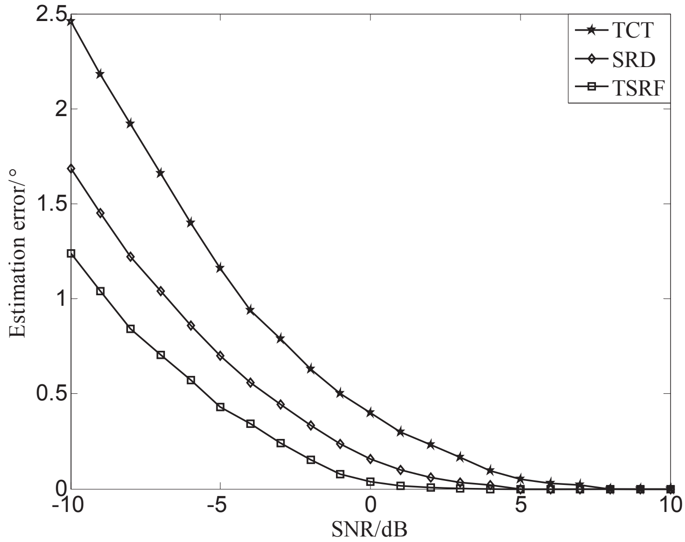

FIGURE 5. Estimation error versus SNR

more accurately than SRD and TCT, as the increase of SNR, they are all convergent at last.

4.3. **Estimation Error versus Number of Snapshots.** Assume that number of snapshots at each frequency varies from 3 to 30, the estimation error is shown in FIGURE 6 when SNR is 4dB. We can see from FIGURE 6, TSRF method can estimate DOA

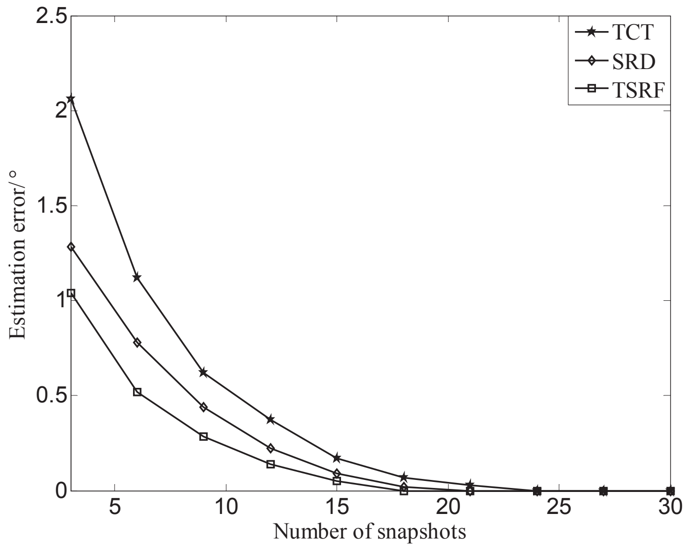

FIGURE 6. Estimation error versus snapshots

more accurately than SRD and TCT, as the increase of number of snapshots, they are all convergent at last.

4.4. **Resolution.** Assume that two far-field wideband signals arrive at the array with same power from $(45°, 59°)$,$(48°, 56°)$, SNR is 5dB, the normalization spectrum are given in FIGURE 7, 8 and 9. We can see from FIGURE 7∼9, when the two signals are near,

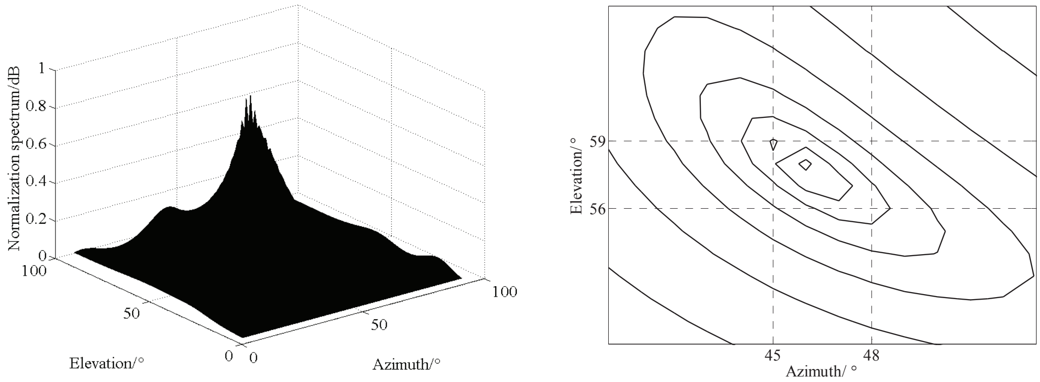

FIGURE 7. Normalization spectrum of TCT for two near signals

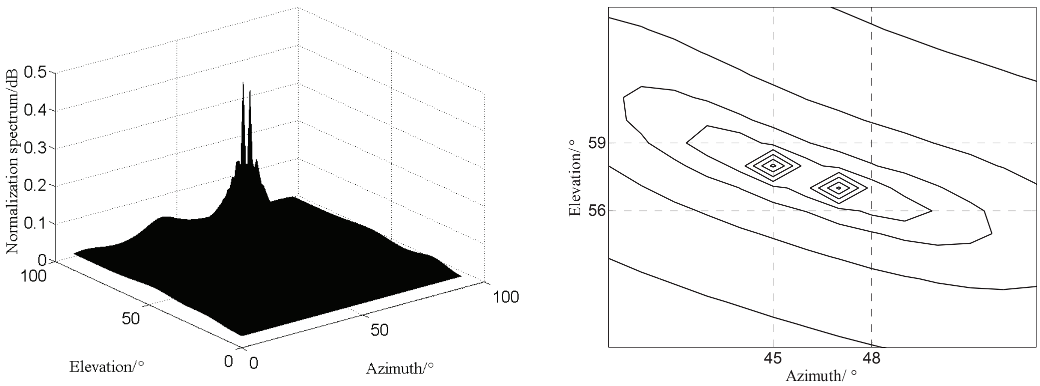

FIGURE 8. Normalization spectrum of SRD for two near signals

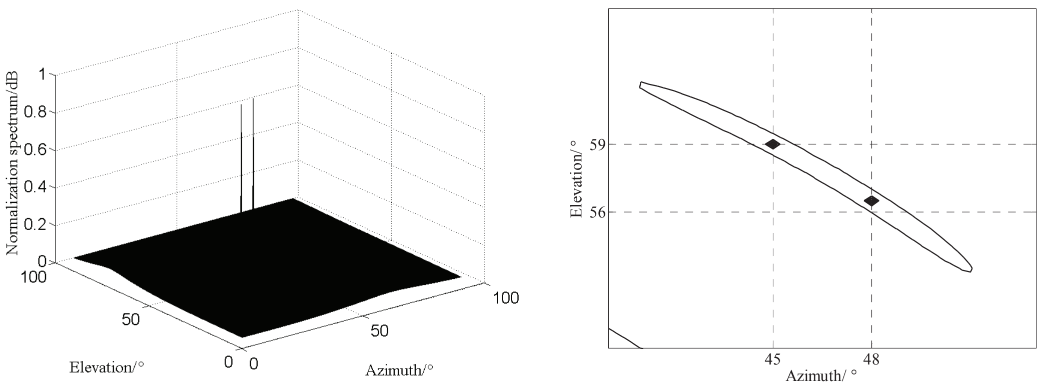

FIGURE 9. Normalization spectrum of TSRF for two near signals

only TSRF can resolve them accurately, but TCT and SRD have failed.

4.5. **Estimation Probability of Success versus Relative Bandwidth.** Suppose SNR is 5dB, the relative bandwidth ranges from 0 to 50% , if the estimation error is less than 0.4, it is thought to be successful, estimation probability of success versus relative bandwidth is shown in FIGURE 10. It is seen from FIGURE 10, when the relative

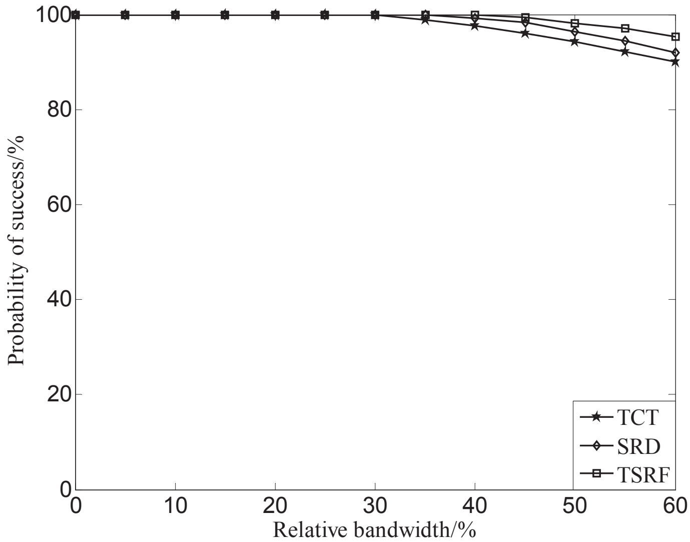

FIGURE 10. Estimation probability of success versus relative bandwidth

bandwidth is less then and equal to 30%, all the methods can estimate DOA successfully, but the performance is deteriorating as the relative bandwidth increases, in contrast, TSRF is the better than SRD and TCT.

5. **Conclusions.** The DOA estimation method for two-dimensional wideband signals based on sparse recovery is proposed in continuous frequency domain in this paper, the iterative functions are constructed and calculated by output signal, after that the support set is acquired according to the root finding and corresponding semidefinite programming, then DOA is acquired by integrating all frequency bins, meanwhile the signal can also be recovered in continuous frequency domain. This method avoids the error brought by sparse recovery in discrete domain, it particularly performs well when the number of snapshots is small and SNR is low.

Acknowledgment. This work is supported by the national natural science fund of China (61501176) and University Nursing Program for Young Scholars with Creative Talents in Heilongjiang Province, (UNPYSCT-2016017).

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