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The Correlation Properties of Urban Traffic Networks

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Abstract. *The correlation property is one of the essential structural statistic features of complex networks. This paper studies Beijing road and taxi traffic networks based on four categories. Firstly, to identify the correlation property of the undirected unweighted Beijing road networks, this paper adopts the Pearson correlation coefficient measure. Secondly, to characterize the correlation property of the unweighted directed taxi traffic networks, we use the node-based in-degree and out-degree correlation and the arc-based degree-degree correlation measures. Thirdly, to describe the correlation property of the weighted directed taxi traffic networks, this paper utilizes the arc-based weighted degree-degree correlation, the arc-based strength-degree correlation and the edge-based strength-degree correlation measures. Finally, to depict the correlation property of the undirected weighted traffic networks, we employ the node-based strength-degree correlation, the node-based strength-strength correlation and the edge-based weighted degree-degree correlation measures. Based on our study, we find that all these correlations are positive. Moreover, this paper aims to explain the experimental results based on reality phenomenon and discover the law of transportation development with the increasement of ring.*
Keywords: Complex network, Traffic network, Undirected unweighted network, Directed weighted network, Degree correlation.

1. **Introduction.** Now, a growing body of research is focusing on the application of complex network theory [1] on various domains [2], such as social networks, electric power grids, subway systems, the Internet, and so on. While the wide application of complex network theory in transportation [3] has opened the door to advancements in this field, such as urban road traffic networks [4], rail networks [5], railway networks [6] and transit

networks [7] and so on. Liu et al. [4] introduced the complex network statistical characteristics and spatial autocorrelation models to the urban-rural road network of Wuhan in China. Latora et al. [5] testified the small-world property of the Boston transportation system, suggesting the topology of the transportation network. Researchers identified that the Chinese railway network is a small world network with the scale-free property with good survivability under random attacks [6]. Rosvall et al. [8] illustrated that the intuitive expectation of most modern planned cities are presumably simple, while historical cities with a complicated past of cut and paste construction are more complex from an information perspective. Reference [9] examined the structural characteristics of the urban road traffic network of Beijing on the basis of a new weighted traffic network model. Fancello et al. [10] advanced an integrated performance indicator in the urban road infrastructure for evaluating network functionality and the impact of transport system interventions. Our previous work [11] modeled the urban road traffic networks and the urban taxi traffic networks of Beijing with two newly proposed modeling methods named TTI and TTIS respectively, and analyzed the essential topological features of the networks based on complex network theory.

Correlation property is a significant structural statistic characteristic of complex networks. Han et al. [12] testified that, in European, four specific airline flight networks are disassortative and the strength-degree correlations of the networks imply that the larger the airport the stronger the capability of managing transportation flux. Researchers [13] demonstrated that each of the four domestic transit networks is assortative with a negative clustering coefficient-degree correlation. Lv et al. [14] investigated the second-order centrality correlation in scale-free networks.

However, relatively few investigations were devoted to other correlation properties of the traffic networks, particularly the urban road traffic networks based on the taxi system. Inspired by existing results for directed weighted networks reported in [15] and the modeling of Beijing road and taxi network models [11] based on the approach in [16] [17], this paper concentrates on the various correlation features of the urban road traffic networks and the urban taxi traffic networks in Beijing, in order to explain the experimental results based on reality phenomenon. Since it is well known that the road network in Beijing is expanding in the form of loop-lines, following the 2nd, 3rd, 4th and 5th ring roads, and therefore this paper also aims to discover the law of transportation development with the increasement of ring.

2. **Degree Correlation of Beijing Road Traffic Networks.**

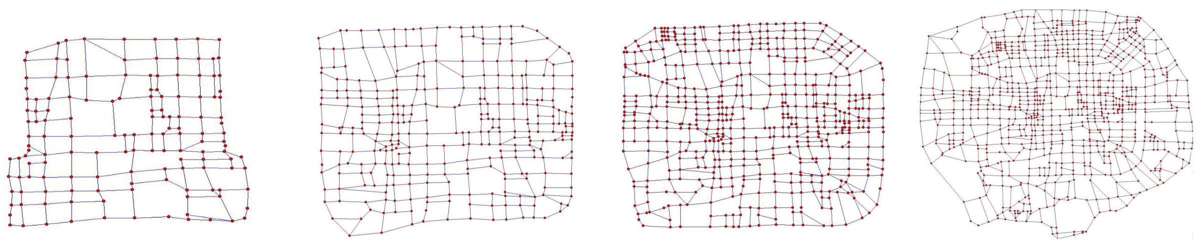
This section focuses on the degree correlations of the undirected unweighted road traffic networks and its variation with the increasement of ring number.

Newman showed an intuitive method to calculate the degree correlation [18]: through an edge, there are two nodes and their degree values, and then we traverse all edges to obtain the two sequences, and analyze the correlation between these two sequences. Newman used the Pearson correlation coefficient to describe the degree correlation of a network. The formula for calculating the Pearson correlation coefficient [18] is given as follows:

$$r = \frac{M^{-1}\sum_{e_{ij}} k_i k_j - [M^{-1}\sum_{e_{ij}} \frac{1}{2}(k_i + k_j)]^2}{M^{-1}\sum_{e_{ij}} \frac{1}{2}\left(k_i^2 + k_j^2\right) - [M^{-1}\sum_{e_{ij}} \frac{1}{2}(k_i + k_j)]^2} \tag{1}$$

where M is the number of edges in the network, k_i is the nodal degree of Node v_i, and e_{ij} means the edge connecting Node v_i and Node v_j. According to the positive or negative of the degree correlation coefficient, networks can be classified into three categories, i.e., assortative, disassortative and neutral networks. In the assortative (disassortative) networks, r is positive (negative) and the high-degree nodes tend to connect to other high-(low-) degree nodes, while the neutral networks do not have such features.

Based on the undirected unweighted Beijing road traffic network models in [11] (see Figure 1, including the road traffic networks of Beijing inside 2nd, 3rd, 4th and 5th ring) and the road map of Beijing within the 5th ring as shown in Figure 2, according to the formula of the Pearson correlation coefficient r, we obtain the Pearson correlation coefficients of Beijing road networks as shown in Table 1. Moreover, we display the degree correlation curves as shown in Figure 3 by MATLAB, while k at degree k_{nn} is the average degree value of neighbors of nodes with degree k.



(a) (b) (c) (d)

FIGURE 1. The road network inside of the each ring of Beijing

Table 1 lists some indices (where APL means the average shortest path length) containing the degree correlations of these networks, and all of the correlation coefficients are positive which implies the trend that the intersections with more roads tend to connect

FIGURE 2. The road map of Beijing within the 5th ring

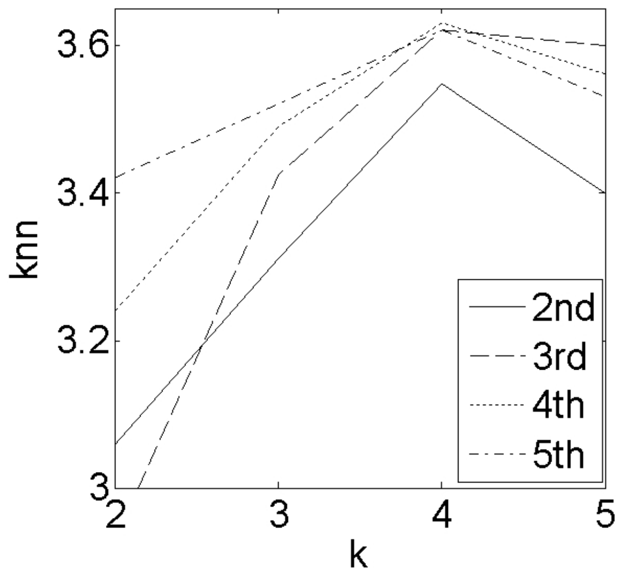


FIGURE 3. Degree correlation properties of Beijing road traffic networks

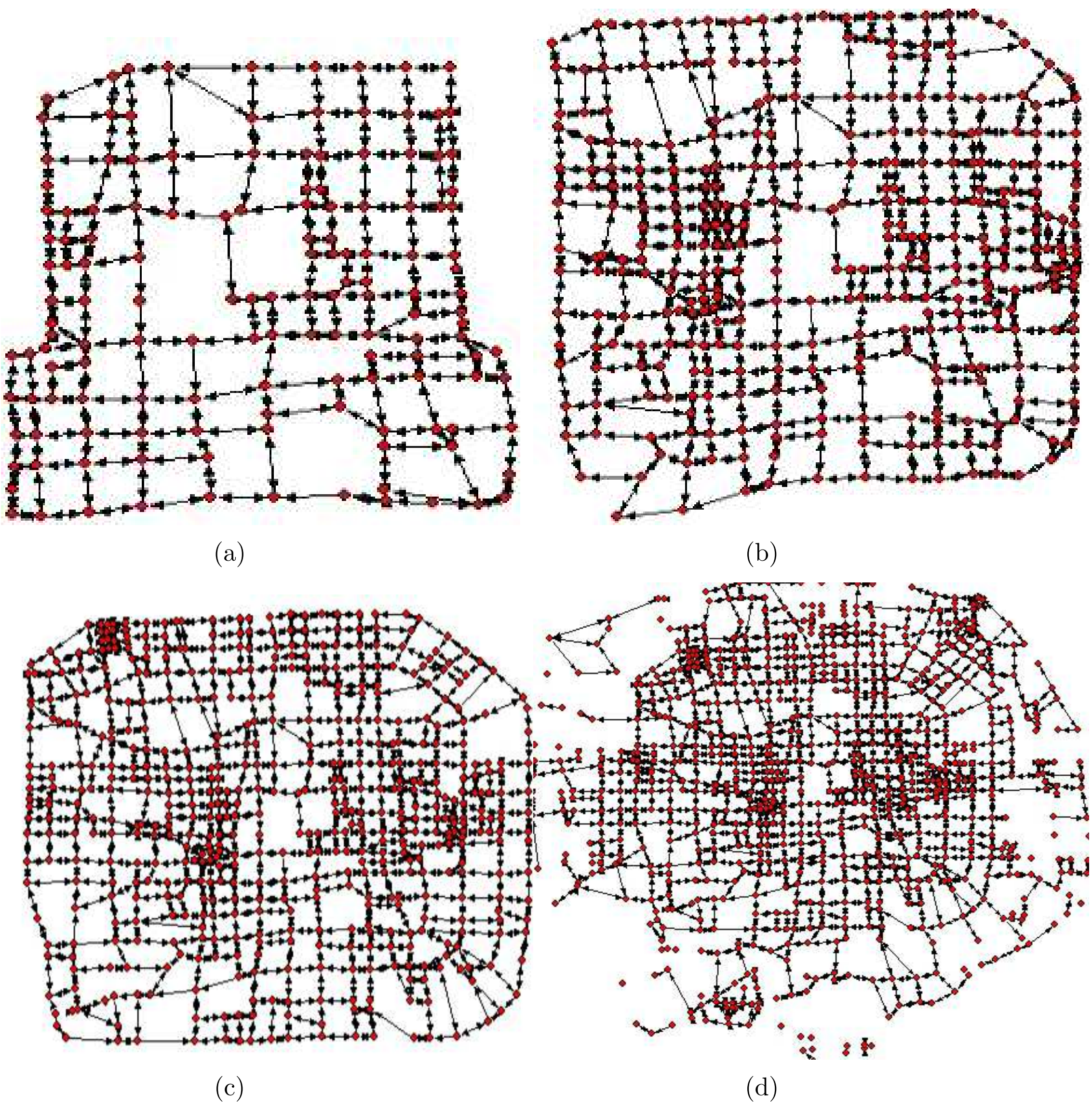


Figure 4. The directed-weighted Beijing taxi traffic networks

Table 1. The Pearson correlation coefficients of Beijing road networks

Range of coverage	Inside the 2nd ring	Inside the 3rd ring	Inside the 4th ring	Inside the 5th ring
N	144	322	547	871
M	235	548	946	1517
r	0.2243	0.2049	0.1258	0.0793
APL	7.7430	10.9585	13.7791	16.5174
Diameter	18	25	32	38

to other intersections with more roads. This conclusion illustrates the spontaneous expansion of urban road networks [19], i.e., two prosperous districts in short distance tend to be connected by a road. While the traffic network with a shorter average shortest path length and a smaller network diameter signifies that the transportation efficiency of the traffic network is lower, and the absolute value of r decreases with the increasement of ring, which denotes that this declining trend with the expansion of ring in Beijing leads to the increasement of the average shortest path length and the network diameter as shown in Table 1. We can infer that the transportation efficiency of Beijing road network is decreasing with the expansion of ring.

Additionally, from Figure 3, we can see that all of the correlation curves are with a positive slope except the end parts of these curves since there are no higher-degree nodes could be linked to the nodes with the maximum degree value of 5, which means that the networks are assortative. And the curves become gentler with the increasement of ring number, which agrees with the phenomenon that observed from Table 1.

3. **Correlation Properties of Beijing Taxi Traffic Networks.** Here are the main results in this paper. This section takes the traffic flow and its direction into consideration in order to investigate the correlation properties for directed weighted traffic networks deeply based on the directed weighted Beijing taxi traffic network models proposed in [11] (see Figure 4, where Figure 4 displays the directed weighted taxi traffic networks of Beijing inside the 2nd, 3rd, 4th and 5th rings, and the direction of taxi flow is denoted by the arrow while the weight of one direction of an edge represents the value of taxi traffic flow in this direction), obtaining the correlation curves by MATLAB as shown in Figure 5 - Figure 12 which are explained in the following.

3.1. **Correlation Properties of Beijing Directed Unweighted Taxi Traffic Networks.** This subsection concentrates on the correlation properties of the directed unweighted Beijing taxi traffic networks, taking only the direction of arcs into consideration, on the basis of the node-based in-degree and out-degree correlation and the arc-based degree-degree correlations to study the correlation features of these Beijing taxi traffic networks separately.

The node-based in-degree and out-degree correlation $k_{vv}(k_{in})$ is defined as [20]:

$$k_{vv}\left(k_{in}\right) = \Big(\sum_{i:k_i^{in}=k_{in}} k_i^{out}\Big) / \left[N \cdot P_{in}\left(k_{in}\right)\right] \tag{2}$$

where $P_{in}(k_{in})$ is the nodal in-degree distribution, ${k_i}^{in}$ and ${k_i}^{out}$ mean the in-degree and out-degree of node v_i respectively, and N is the total number of nodes. We draw the correlation curves of the directed unweighted networks in Figure 5 on the basis of Eq.(2) by MATLAB.

From Figure 5, we observe that these curves appear a nearly positive slope, which implies that the intersection with more in-streets is likely to possess more out-streets. Furthermore, Table 2 lists the proportion of one-way and two-way roads of Beijing taxi networks, and we observed that a large proportion of roads are two-way. It seems that most of the roads are bi-directional, which can account for the experimental phenomenon that the intersection with more in-streets may well own more out-streets.

Table 2. The proportion of different roads of Beijing taxi networks

Range of coverage	Inside the 2nd ring	Inside the 3rd ring	Inside the 4th ring	Inside the 5th ring
One-way roads	3.004	2.390	5.184	9.563
Two-way roads	96.996	97.610	94.816	90.437

The arc-based degree-degree correlations focus on the correlations between the in(out)-degree of the head node and the in(out)-degree of the tail node for each arc. Obviously,

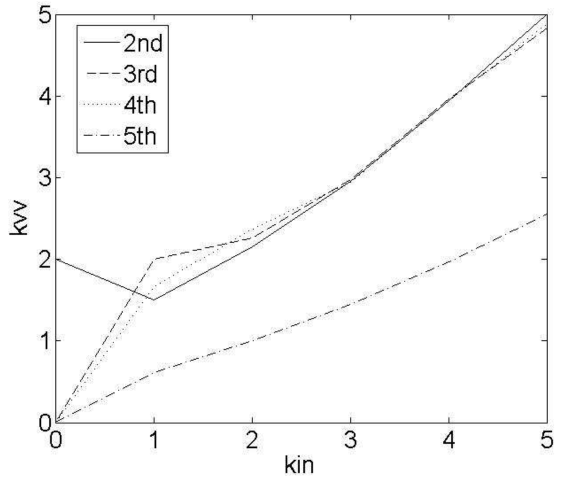


FIGURE 5. The node-based in-degree and out-degree correlation of Beijing taxi traffic networks

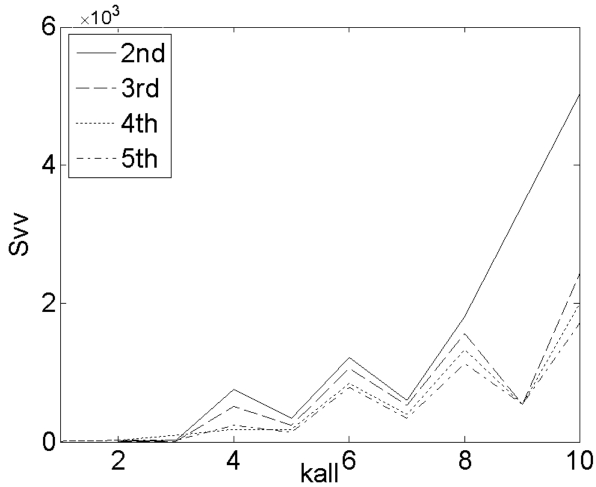
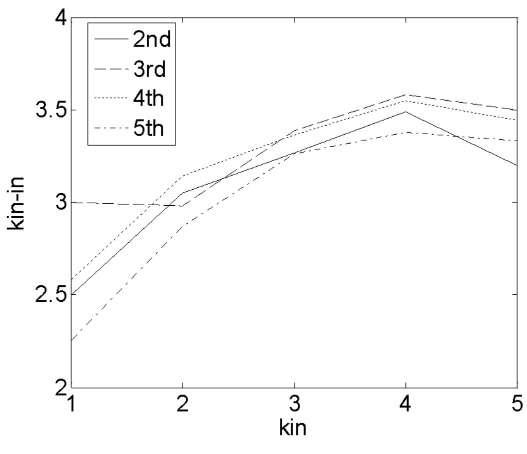


FIGURE 6. The node-based strength-degree correlations of Beijing taxi traffic networks

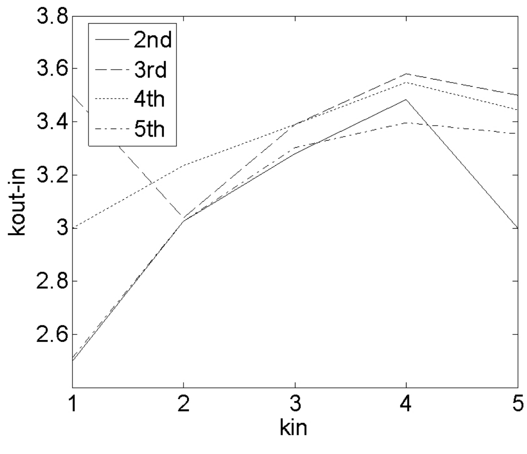
there are four combinations, i.e., in-in, out-in, in-out, out-out correlations. For example, the out-in degree-degree correlation [20] can be denoted as $k_{out-in}(k_{in})$ as follows.

$$k_{out-in}\left(k_{in}\right) = \left(\sum_{i:k_i^{in}=k_{in}} \frac{1}{k_i^{in}} \sum_{j=1}^{N} a_{ji} k_j^{out}\right) / [N \cdot P_{in}(k_{in})] \tag{3}$$

Based on these definitions, two arc-based correlation curves of the directed unweighted Beijing taxi networks are shown in Figure 7.



(a)



(b)

FIGURE 7. The arc-based correlations of Beijing taxi traffic networks

Figure 7 shows two examples of four arc-based degree-degree correlations, $k_{in-in}(k_{in})$ and $k_{out-in}(k_{in})$ (similar results can be obtained for $k_{in-out}(k_{out})$ and $k_{out-out}(k_{out})$), we can observe that these curves are almost all with a positive slope, which infers that the intersection with more in(out)-streets tends to be linked to the intersection with either more in-streets or out-streets.

3.2. **Correlation Properties of Beijing Undirected Weighted Taxi Traffic Networks.** This subsection ignores the impact of flow direction on each edge. Thus, there are three correlations to study the feature of undirected weighted networks, i.e., the node-based strength-degree correlation, the node-based strength-strength correlation, and the edge-based weighted degree-degree correlation.

First, the formula of the node-based strength-degree correlation is given as follows:

$$S_{vv}(k) = (\sum_{i:k_i=k} S_i)/[N \cdot P(k)] \tag{4}$$

where S_i denotes the strength of node v_i, and $P(k)$ is the nodal degree distribution. And the corresponding correlation curves are shown in Figure 6, where k_{all} means the sum of nodal in-degree and out-degree.

From Figure 6, we can see that these curves are upward when ignoring the odd k_{all} or only taking the odd k_{all} into account, while the curves are falling from even to odd, upward from odd to even. These performances imply that no matter whether the intersection possesses a one-way road or not, the intersection in possession of more streets tends to be linked to busier intersection, where a busier intersection means that this intersection bears larger traffic flow. The intersection with a one-way road is more likely to connect to a less busier intersection than the intersection without a one-way road, which indicates that the traffic department can artificially set a one-way street to direct the large traffic flow to an adjacent and less congested area or intersection in order to relieve traffic congestion when a region or intersection gets caught in a traffic jam. This result testifies that one-way streets are effective to alleviate traffic congestions [21] from the perspective of complex network theory.

Second, the formula of the node-based strength-strength correlation is given as follows:

$$S_{nn,i} = (\sum_j a_{ij} w_{ij} S_j)/S_i \tag{5}$$

$$S_{nn}(S) = (\sum_{i:S_i=S} S_{nn,i})/[N \cdot P(S)] \tag{6}$$

where w_{ij} represents the weight between nodes v_i and v_j, a_{ij} is the element of the adjacency matrix, and $P(S)$ is the nodal strength distribution. Based on this definition, we obtain the corresponding correlation curves as shown in Figure 8 by MATLAB.

From Figure 8, we can see that all the curves are upward overall, which implies that a busier intersection tends to be linked to other busier intersections.

Last, the formula of the edge-based weighted degree-degree correlation is defined as follows:

$$k_{w\_nn,i} = (\sum_{j \in N_i} w_{ij} k_j)/S_i \tag{7}$$

$$k_{w\_nn}(k) = (\sum_{i:k_i=k} k_{w\_nn,i})/[N \cdot P(k)] \tag{8}$$

and the corresponding correlation curves are shown in Figure 9.

From Figure 9, we can see that all curves are upward overall, which indicates that a busier road tends to connect to an intersection with more roads.

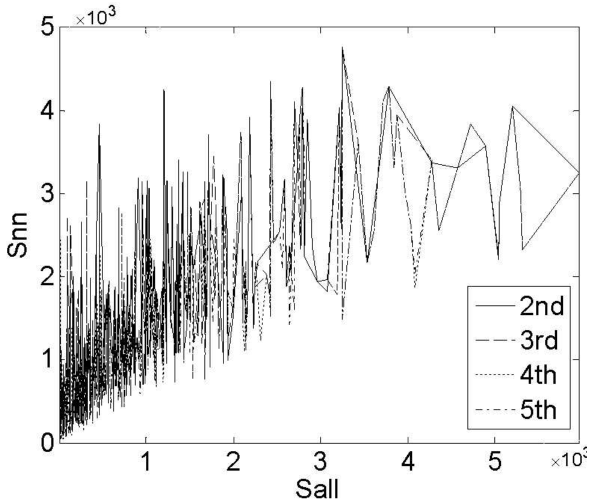


FIGURE 8. The node-based strength-strength correlations of Beijing taxi traffic networks

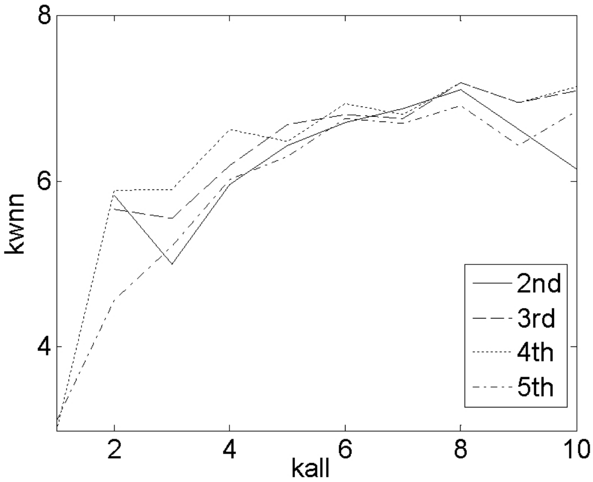


FIGURE 9. The edge-based weighted degree-degree correlations of Beijing taxi traffic networks

3.3. **Correlation properties of Beijing directed weighted taxi traffic networks.** This subsection takes both the weight and direction of arcs into account. There are three correlations used in this analysis of the correlation features of the directed weighted Beijing taxi networks as well, i.e., arc-based weighted degree-degree correlation, arc-based strength-degree correlation and edge-based strength-degree correlation [15].

First, there are four arc-based weighted degree-degree correlations, i.e., in-in, out-in, in-out, out-out correlations. For example, the out-in one can be denoted as $k_{w\_out-in}(k_{in})$, which is defined as follows:

$$k_{w\_out-in,i} = (\sum_{j=1}^{N} a_{ji} w_{ji} k_j^{out})/S_i^{in} \tag{9}$$

$$k_{w\_out-in}(k_{in}) = (\sum_{i:k_i^{in}=k_{in}} k_{w\_out-in,i})/[N \cdot P_{in}(k_{in})] \tag{10}$$

and the corresponding correlation curves are shown in Figure 10.

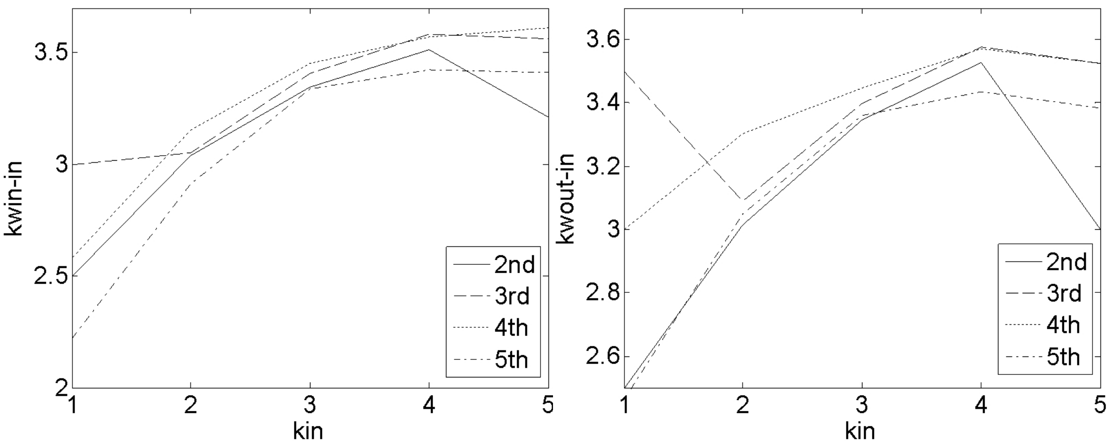
Figure 10 shows two examples of four arc-based weighted degree-degree correlations, $k_{w\_in-in}(k_{in})$ and $k_{w\_out-in}(k_{in})$ (similar results can be obtained for $k_{w\_in-out}(k_{out})$ and $k_{w\_out-out}(k_{out})$). From Figure 10, we can realize that all these curves are with a positive slope, which indicates that the road with larger traffic flow tends to connect to the intersection with more roads, which is consistent with the conclusion from Figure 9.

Second, there are four arc-based strength-degree correlations also, i.e., in-in, out-in, in-out, out-out correlations. The out-in one can be denoted as $S_{out-in}(k_{in})$, and the formula is:

$$S_{out-in}(k_{in}) = (\sum_{i:k_i^{in}=k_{in}} S_i^{out})/[N \cdot P_{in}(k_{in})] \tag{11}$$

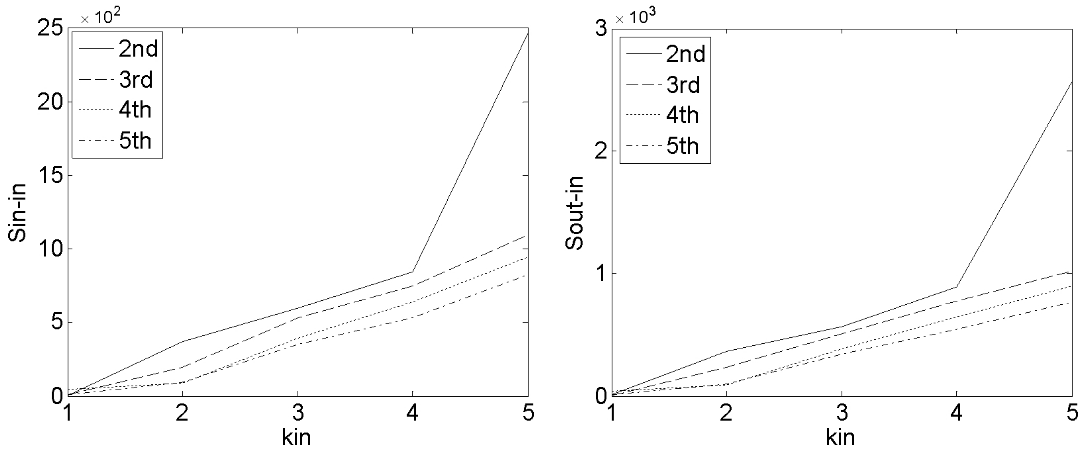
and the corresponding correlation curves are shown in Figure 11.

Figure 11 shows two examples of four arc-based strength-degree correlations, $S_{in-in}(k_{in})$ and $S_{out-in}(k_{in})$ (similar results can be obtained from $S_{in-out}(k_{out})$ and $S_{out-out}(k_{out})$).



(a) (b)

FIGURE 10. The arc-based weighted degree-degree correlations of Beijing taxi traffic networks



(a) (b)

FIGURE 11. The arc-based strength-degree correlations of Beijing taxi traffic networks

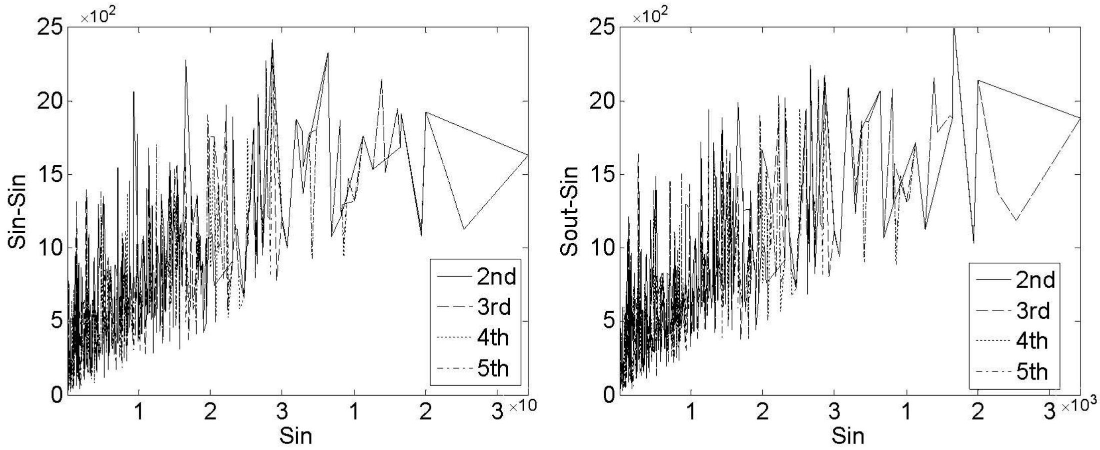
From Figure 11, we find that these curves all have a positive slope. This implies that the intersection linked with more streets is likely to bear larger traffic flow.

Last, there are four arc-based strength-strength correlations as well, i.e., in-in, out-in, in-out, out-out correlations. The out-in correlation can be denoted as $S_{out-in}(S_{in})$, and the formula is:

$$S_{out-in}(S_{in}) = (\sum_{i:S_i^{in}=S_{in}} \frac{1}{S_i^{in}} \sum_{j=1}^{N} a_{ij} w_{ij} S_j^{out}) / [N \cdot P_{in}(S_{in})] \tag{12}$$

while the corresponding correlation curves are shown in Figure 12.

Figure 12 displays two examples of four arc-based strength-strength correlations, $S_{in-in}(S_{in})$ and $S_{out-in}(S_{in})$ (similar result can be obtained from $S_{in-out}(S_{out})$ and $S_{out-out}(S_{out})$).



(a) (b)

FIGURE 12. The arc-based strength-strength correlations of Beijing taxi traffic networks

From Figure 12, we can see that these curves are all with a positive slope, which indicates that a busier intersection tends to connect with other busier intersections, which is consistent with the conclusion from Figure 8.

4. **Conclusions.** This paper computes and analyzes the correlation features of the undirected unweighted road networks and the directed weighted taxi traffic networks inside each ring of Beijing city by MATALB. And the specific conclusions are as follows.

(1) Beijing traffic networks are assortative.
(2) The transportation efficiency of Beijing road network decreases with the expansion of ring.
(3) The intersection with more in-streets is likely to own more out-streets because of the large proportion of two-way roads.
(4) The intersection with more in(out)-streets tends to be linked to the intersection with either more in-streets or out-streets.
(5) The intersection with more streets tends to be linked to other busier intersections.
(6) The intersection with one-way roads is likely to connect with a less busy intersection, which is consistent with the goal of setting one-way street that is to relieve traffic congestion.
(7) The busier intersection tends to be connected to a busier intersection.
(8) The busier road tends to connect to a intersection with more roads.
(9) The intersection with more streets is likely to be busier.

It is evident that the correlation curves are all positive. And it is significant that a node with a larger (in- , out- or all-)degree(strength) linked to other nodes with a larger (in- , out- or all-)degree(strength) would advance the transportation efficiency of the network, while this may be the main reason why these correlations of the traffic networks are all positive.

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