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Generalized Locality Preserving Projection for Multimodal Biometric Recognition

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Abstract. *Multimodal biometric recognition is a promising personal identity authentication technology which can remedy the limitation of the traditional identity authentication and the unimodal biometrics. Comparing with other three fusion levels of multimodal biometrics, feature level fusion can reduce the redundant information to avoid calculation consumption and acquire the discriminative information to improve the system performance. Complex fusion is a novel feature fusion pattern which takes two features as real part and imaginary part of a complex vector. However, the existing linear methods of complex fusion cannot consider the nonlinear factor. Meanwhile the computations of the nonlinear methods is too great. This paper extended LPP into the complex field and proposed generalized locality preserving projection (GLPP) which takes advantage of the optimal linear approximations to find the nonlinear manifold structures.Face and palm are taken as the experimental objects to conduct the fusion features. Experimental result shows the proposed algorithm achieves much better performance than two unimodal biometrics and other four conventional multimodal biometric algorithms.*
Keywords: Multimodal biometric recognition, Feature fusion, Complex field, LPP

1. **Introduction.** The basic theory of subspace analysis is aimed to find a linear or nonlinear transformation to compress or convert the original data into a low dimensional space. Principal component analysis (PCA) and Fisher discriminant analysis (FDA) are two popular methods. Nevertheless, both PCA and FDA effectively pay attention only to the Euclidean structure of sample space and fail to discover the underlying structure of samples [1], such as face. Locality preserving projection (LPP) finds an embedding that preserves local information, and obtains a locality preserving subspace that best detects the essential sample manifold structure. In the application of face recognition, LPP encodes more discriminating information in the low dimensional face subspace by preserving local structure which is more important than the global structure for classification. So, it significantly outperforms Fisherface and Eigenface which perform FDA and PCA on face recognition respectively .

The above algorithms are applied on unimodal biometric recognition which has drawn extensive attention during the past decades for its huge potentials in many applications. However, the performances of unimodal biometric systems have to contend with a variety of problems such as background noise, signal noise and distortion, and environment or device variations. Therefore, multimodal biometric systems are proposed to solve the above mentioned problems[2, 3]. Along with the fusion in pixel level, score level and decision

level, one important branch is to perform fusion in feature level which can derive the most discriminative information from original multiple feature sets and eliminate the redundant information resulting from the correlation between different feature sets. In general, there are two basic modes for feature level fusion: serial rule and weighted sum rule. However, the former would lead to more redundancy and increase computation cost owing to the high dimensional fusion features. For the latter, the proper determination of weights is a contentious issue. Then Yang generalized PCA and FDA into complex field to produce two new methods for multimodal biometrics: Generalized PCA (GPCA)[4] and Generalized FDA(GFDA) [5], which keep still the original characteristic attained from the real field. An efficient subspace learning algorithm should be able to discover the nonlinear structure of the sample space. Kernel PCA(KPCA) and kernel FDA (KFDA) are two kernel based typical methods in subspace analysis. Whereafter, generalized KPCA(GKPCA) [6] and generalized KFDA(GKFDA)[7] are proposed as the classifier of the complex field for multimodal biometric recognition. However, the computation of them are too expensive. This paper extended LPP into the complex field and proposed generalized locality preserving projection (GLPP) for multimodal biometric recognition. Face and palm are used as two distinct biometric subjects to test our algorithm. The key characteristics are as follows: (1) Two distinct feature sets are fused by a complex vector to construct a unitary space; (2) GLPP is used to resolve the classification problem of the unitary space and ensure the availability of the proposed scheme; (3) Gram-Schmidt transformation is imported to guarantee the orthogonality of eigenvectors in GLPP; (4) GLPP takes advantage of the optimal linear approximations to the eigenfunctions of the Laplace Beltrami operator to find the nonlinear manifold structures. It is superior to the linear methods (GPCA and GFDA) and the nonlinear methods (GKPCA and GKFDA).

The rest of the paper is organized as follows: section 2 introduces the typical fusion pattern and further focuses on the linear and nonlinear approaches of complex fusion; Our algorithm is presented in section 3; In section 4, experimental results are illustrated. Finally, section 5 concludes this paper.

2. **Related Works.**

Multimodal biometric technology is divided into four levels: pixel level, feature level, score level and decision level. Comparing with other three fusion levels, feature level can reduce the redundant information to avoid calculation consumption, and simultaneously acquire the discriminative information to improve the system performance. This section summarizes the current fusion patterns in feature level, and then focuses on linear and nonlinear complex fusion pattern.

2.1. **Fusion Pattern.**

In general, there are two basic modes for feature level fusion: serial rule and weighted sum rule [8]. The former connects two feature vector into a longer fusion vector. This rule consumes large computational resources. For weighted sum rule, the fusion feature is the sum of the two unimodal features multiplied by the respective weighted value. So two problems should be considered: weighted value and dimension. The choice of weights value is no good way, usually in accordance with the experience. If the dimension of two unimodal feature vector is deferent, two ways can be selected: truncating the long vector or filling the short vector by zeros. Besides, Yang [4, 5] proposed a novel method which avoid the large amount of computation and the selection of the weighted value. This method takes two features as real part and imaginary part of a complex vector.

Suppose a and b be two feature vectors derived from two biometric modes respectively, the fusion feature is obtained as $f = a + ib$ where i denotes imaginary unit. So we find complex fusion also need to face the dimension problem. Yang unified dimension during

feature extraction using PCA and KPCA [5]. Normally, the fusion features obtained by serial rule and weighted sum rule are directly used to match. Complex fusion implements feature selection to derive the most discriminative information of fusion feature before matching. GPCA and GFDA are the earliest literature in this area. Whereafter, other methods based on complex fusion are proposed[6, 7].The following part of this section introduces these methods which are divided into linear and nonlinear style.

2.2. **Linear Style.** In a complex field $\mathbb{C}$, the inner product is defined[5] by

$$< x, y >= x^H y \tag{1}$$

where $x, y \in \mathbb{C}$ and H is the denotation of conjugate transpose. We can be easily proved that the inner product meets the following conditions:

- $< x, y >= \overline{< y, x >}$
- $< x, x >\geq 0$, where $< x, x >= 0$ if and only if $x = 0$
- $< k_1 x_1 + k_2 x_2, y >= k_1 < x_1, y > +k_2 < x_2, y >, \forall x_1, x_2, y \in \mathbb{C}$,where k_1, k_2 are two real constants.

So the subspace is a unitary space. Suppose there are a total of n classes, each class of m samples. In the unitary space, the within-class scatter matrix S_w, the between-class scatter matrix S_b and the total scatter matrix S_t can be defined as follows:

$$\begin{array}{rcl} S_w & = & \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \\ S_b & = & \sum_{i=1}^{m} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^H \\ S_t & = & \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H \end{array} \tag{2}$$

where $\bar{x}_i$ and $\bar{x}$ denote the mean vector of class i and the mean of all training samples respectively.

According to the above equations, S_w, S_b and S_t are all non-negative definite Hermite matrices. So, each eigenvalue of the above three matrices is a real number. This conclusion ensures the progress of GPCA because the selection of the principal component is sorted according to the size of the characteristic value. The objective function of GPCA is expressed[4] as

$$J_{GPCA} = \arg\max_{W} |W^H S_t W| \tag{3}$$

where W is the projection matrix which maps the fusion features to a feature space with maximal irrelevant. In the actual operation, the eigenvectors corresponding to the eigenvalues in descending order compose the matrix W. Then the mapping feature g of the fusion feature f is obtained by $g = W^H f$. GFDA is also proposed on the basis of GPCA similar to the real field. Yang used GFDA in face recognition and named as combined fisherface. The objective function of this method is displayed[5] as follows:

$$J_{GFDA} = \arg\max_{W} \frac{|W^H S_b W|}{|W^H S_w W|} \tag{4}$$

where W is composed by the generalized eigenvectors of the generalized eigenequation $S_b X = \lambda S_w X$ corresponding to the several largest eigenvalues. Then the mapping feature g can be computed in the same way as GPCA.

2.3. **Nolinear Style.** In real field, nonlinear problem is usually converted to a linear problem in a high-dimensional space. Kernal method uses a nonlinear transformation ϕ to convert the nonlinear problem into a linear problem of high dimensional space. First

of all, a nonlinear mapping ϕ is used to map the sample space R^d into the nonlinear space F:

$$\begin{array}{rccc} \phi: & R^d & \rightarrow & F \\ & x & \mapsto & \phi(x) \end{array} \tag{5}$$

Suppose there are a total of n classes, each class of m training samples. The within-class scatter matrix $\tilde{S}_w$, the between-class scatter matrix $\tilde{S}_b$ and the total scatter matrix $\tilde{S}_t$ are constructed in the nonlinear space F:

$$\begin{array}{rcl} \tilde{S}_w & = & \sum_{i=1}^{n}\sum_{j=1}^{m}(\phi(x_i^j)-\bar{\phi}_i)(\phi(x_i^j)-\bar{\phi}_i)^H \\ \tilde{S}_b & = & \sum_{i=1}^{m}(\bar{\phi}_i-\bar{\phi})(\bar{\phi}_i-\bar{\phi})^H \\ \tilde{S}_t & = & \sum_{i=1}^{n}\sum_{j=1}^{m}(\phi(x_i^j)-\bar{\phi}_i)(\phi(x_i^j)-\bar{\phi}_i)^H \end{array} \tag{6}$$

where $\bar{\phi}_i = \frac{1}{m}\sum_{j=1}^{m}\phi(x_i^j)$, $\bar{\phi} = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\phi(x_i^j)$. However, it is difficult to do so directly because of the high dimension of the nonlinear space F. Fortunately, kernel tricks can avoid this computation by the following rule

$$k(x_i^j, x_s^t) =< \phi(x_i^j), \phi(x_s^t) >= \phi(x_i^j)^H\phi(x_s^t) \tag{7}$$

where $k(\cdot)$ denotes the kernel function of two arbitrary examples x_i^j and x_s^t . In this way, the explicit mapping ϕ is not required.

GKPCA and GKFDA are two representatives of the nonlinear methods in complex field. Define the matrix $Q = [\phi(x_1^1)-\bar{\phi}, \phi(x_1^2)-\bar{\phi}, \cdots, \phi(x_1^m)-\bar{\phi}, \cdots, \phi(x_n^m)-\bar{\phi}]$, $\tilde{S}_t = QQ^H$. The objective function of GKPCA is expressed[6] as follows:

$$J_{GKPCA} = \arg\max_{W} |W^H\tilde{S}_tW| \tag{8}$$

Singular value decomposition (SVD) technique is adopted to reduce computational effort derived from the high dimensional nonlinear space F, so $\tilde{R} = Q^HQ$ is obtained by kernel function, and then centralize $\tilde{R}$ as

$$R = \tilde{R} - 1_{nm}\tilde{R} - \tilde{R}1_{nm} + 1_{nm}\tilde{R}1_{nm} \tag{9}$$

where 1_{nm} denotes a $nm \times nm$ matrix whose elements are all equal to $1/nm$.

GKPCA makes use of the between-class information, but GKFDA also considers the within-class information. Imitating KFDA of the real field, the objective function of GKFDA is expressed [7] as

$$J_{GKFDA} = \arg\max_{W} \frac{|W^H\tilde{S}_bW|}{|W^H\tilde{S}_wW|} \tag{10}$$

then W is the matrix composed by the generalized eigenvectors of the generalized eigenequation $\tilde{S}_bX = \lambda\tilde{S}_wX$ corresponding to the several largest eigenvalues. According to the theory of reproducing kernel and the theoretical formulae computation, the function $\tilde{S}_bX = \lambda\tilde{S}_wX$ can be converted into $K_bX = \lambda K_wX$ where

$$\begin{array}{rcl} K_b & = & \frac{1}{n}\sum_{i=1}^{n}(u_i-u)(u_i-u)^H \\ K_w & = & \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}(\alpha_i^j-u_i)(\alpha_i^j-u_i)^H \end{array} \tag{11}$$

where

$$\begin{array}{rcl} u_i & = & [\frac{1}{m}\sum_{j=1}^{m}k(x_1^1,x_i^j), \cdots, \frac{1}{m}\sum_{j=1}^{m}k(x_n^m,x_i^j)] \\ u & = & [\frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}k(x_1^1,x_i^j), \cdots, k(x_n^m,x_i^j)] \\ \alpha & = & [k(x_1^1,x_i^j), \cdots, k(x_n^m,x_i^j)] \end{array} \tag{12}$$

So, W is the solution of the generalized eigenequation $K_bW = \lambda K_wW$. The remainder operation is same with GKPCA.

3. **Proposed algorithm.**

3.1. **Background.** LPP was originally proposed as a manifold learning algorithm in order to study and analyze the nonlinear manifolds. Manifold learning attempts to find the low dimensional manifold from the high dimensional manifold and determine the corresponding embedding projection to realize dimensionality reduction or data visualization. This operation is more superior than the traditional methods such as PCA and multi-dimensional scaling (MDS). Manifold learning can be roughly divided into two categories: one is the global approach which calculates the relationship for each point with all the other points and establishes a fully connected graph, for example Isometric Mapping(Isomap); The other is the local approach that considers the relationship between each point and its neighborhood points and defines the edges of the graph usually by k-nearest neighbor algorithm or ϵ-nearest neighbor algorithm, such as Locally Linear Embedding (LLE) and Laplacian Eigenmap (LE).

Comparing with Isomap, LLE has several advantages including faster optimization when implemented to take advantage of sparse matrix algorithms[9]. However, the low dimensional embedding of LLE does not keep a distance relationship.Furthermore, least squares problem faces the parameter selection during calculating the reconstruction weight. The different parameter will cause the different reconstruction weight which affects the final embedding results. Comparing with LLE, the weight selection of LE is more direct and simpler[10], which sets the weight directly without solving a linear equations. Because of this, LE is the fastest in the typical manifold learning method, but is not suitable to recover the low dimensional structure same with LLE. Similarly, LE is also sensitive to noise and outliers.

In order to improve generalization learning ability of LE, He[1] proposed LPP which takes the mapping from the original high dimensional space to the low dimensional space as a linear projection. This method is actually the linear approximation of LE. LPP utilizes the projection matrix to transform the implicit nonlinear mapping of LE into the explicit linear mapping, and a new sample can obtain the projection points in the embedding space by the projection matrix directly. Different from the traditional linear subspace method such as PCA and FDA, LPP optimally preserves local neighborhood relationship during projection and finds the nonlinear manifold.

3.2. **Generalized Locality Preserving Projection.** Subspace analysis is widely used in pattern recognition. This kind of methods realize dimensionality reduction mostly by starting from the global feature. Manifold learning usually considers the local feature, which presents a new tool for pattern recognition. For face recognition, it is proved LPP outperforms PCA and FDA [1]. Furthermore, LPP takes advantage of the projection matrix to realize the dimensionality reduction similar with PCA and FDA. Imitating the example of GPCA and GFDA, this paper proposed generalized locality preserving projection(GLPP) to extend LPP into the complex field and apply to multimodal biometrics. Furthermore, Gram-Schmidt orthogonalization is added to ensure the orthogonality of the eigenvectors to improve the performance of the algorithm.

For a given training matrix $X = [x_1, x_2, \cdots, x_n]$ where $x_i \in \mathbb{C}^p (i = 1, 2, \cdots, n)$ is a p-dimensional column vector. GLPP seeks a projection matrix W to gain $y_i = W^H x_i$ which represents x_i in a lower dimension feature space $\mathbb{C}^d (d < p)$ as much as possible, and also makes the adjacent points as close as possible. The objective function to select the projection matrix W is as follows:

$$min \sum_{ij} (y_i - y_j)^2 s_{ij} = min \sum_{ij} (W^H x_i - W^H x_j)^2 s_{ij} \tag{13}$$

where s_{ij} denotes the similarity between x_i and x_j. If $||x_i - x_j||^2 < \varepsilon$ where $\varepsilon > 0$ is sufficiently small, the neighbor relationship will be established between x_i and x_j and s_{ij} is equal to nonzero; Otherwise, $s_{ij} = 0$. Then, s_{ij} can be defined in detail by two ways:

$$s_{ij} = \begin{cases} exp(-||x_i - x_j||^2/t), & ||x_i - x_j||^2 < \varepsilon \\ 0, & otherwise \end{cases} \tag{14}$$

or

$$s_{ij} = \begin{cases} 1, & ||x_i - x_j||^2 < \varepsilon \\ 0, & otherwise \end{cases} \tag{15}$$

In a unitary space, the distance between the complex vectors x_i and x_j is defined as follows:

$$||x_i - x_j|| = \sqrt{(x_i - x_j)^H (x_i - x_j)} \tag{16}$$

let $S = (s_{ij})_{n\times n}$, it is obviously a symmetric matrix. Following some simple algebraic steps, we see that

$$\begin{aligned} &\textstyle\sum_{ij}(W^H x_i - W^H x_j)^2 s_{ij} \\ &= \textstyle\sum_{ij}(W^H x_i - W^H x_j)(W^H x_i - W^H x_j)^H s_{ij} \\ &= 2[\textstyle\sum_{ij} W^H x_i s_{ij} x_i^H W - \sum_{ij} W^H x_i s_{ij} x_j^H W] \\ &= 2[\textstyle\sum_{i} W^H x_i D_{ii} x_i^H W - W^H X S X^H W] \\ &= 2[W^H X D X^H W - W^H X S X^H W] \\ &= 2W^H X(D - S)X^H W \\ &= 2W^H X L X^H W \end{aligned} \tag{17}$$

The coefficient '2' is ignored. $L = D - S$ is a laplace matrix, and $D = (D_{ij})_{n\times n}$ is a diagonal matrix where $D_{ij} = \begin{cases} \sum_{k=1}^{n} S_{ik}, & i = j \\ 0, & i \neq j \end{cases}$. D can be taken as a measure on the data points. The bigger the value D_{ii} is, the more 'important' y_i is. So, in order to remove the scaling issue and the translation arbitrariness, we impose a constraint as follows:

$$\sum_i D_{ii} y_i^2 = 1 \Longrightarrow W^H X D X^H W = 1 \tag{18}$$

Therefore, the minimum problem (equation (13)) converts into

$$\arg\min_{W^H X D X^H W = 1} W^H X L X^H W \tag{19}$$

So, the projection matrix W is attained by the minimum eigenvalue solution to the generalized eigenvalue problem:

$$X L X^H W = \lambda X D X^H W \tag{20}$$

Note that D and S are both symmetric and real matrices, then $L = D - S$ is also a symmetric real matrix. It can be further proved that XLX^H and XDX^H are hermitian matrices. The eigenvalue of equation (20) are all real number. This conclusion ensures GLPP can select the eigenvector according to the size of the corresponding eigenvalue to compose the projection matrix W.

However, the symmetry of the solution matrix $(XDX^H)^{-1}XLX^H$ cannot be guaranteed, the eigenvector cannot guarantee its orthogonality. So Gram-Schmidt orthogonalization is imported to convert the eigenvectors into orthogonal vectors. Suppose $\lambda_1 < \lambda_2 < \cdots < \lambda_k$ are the first k largest eigenvalues of the matrix $(XDX^H)^{-1}XLX^H$ in

order of size, and $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$ are the corresponding eigenvectors, the process of orthogonal transformation is as follows:

$$\begin{aligned} w_1 &= \alpha_1 \\ w_2 &= \alpha_2 - \tfrac{(\alpha_2, w_1)}{(w_1, w_1)} w_1 \\ w_3 &= \alpha_3 - \tfrac{(\alpha_3, w_1)}{(w_1, w_1)} w_1 - \tfrac{(\alpha_3, w_2)}{(w_2, w_2)} w_2 \\ &\vdots \\ &\vdots \\ w_k &= \alpha_k - \tfrac{(\alpha_k, w_1)}{(w_1, w_1)} w_1 - \tfrac{(\alpha_k, w_2)}{(w_2, w_2)} w_2 - \cdots - \tfrac{(\alpha_k, w_{k-1})}{(w_{k-1}, w_{k-1})} w_{k-1} \end{aligned} \tag{21}$$

Finally, the orthogonal projection matrix is $W = [w_1, w_2, \cdots, w_k]$. Then the ith projection feature vector $y_i = W^T x_i$ can be obtained.

3.3. **Connections to GPCA.** This section presents a theoretical analysis of GLPP and its connections with GPCA. We adopt the same hypothesis with LPP in the real field [1]. The parameter ε is used to defined the weight matrix S where ε represents the locality between x_i and x_j. If we take ε to be sufficiently small, we preserve the local structure which is the aim of GLPP. If ε is infinity, any two samples x_i and x_j can be considered that they have a neighbor relationship. So, suppose $s_{ij} = \frac{1}{n^2}(\forall i, j)$, then $D_{ii} = \sum_j s_{ji} = \frac{1}{n}$. Let I be the identity matrix, and e is a column vector taking one at each entry, then the laplacian matrix $L = D - S = \frac{1}{n} I - \frac{1}{n^2} e e^T$ where the transpose T is equivalent to the conjugate transpose H because L is the real matrix, it can be proved that XLX^H is the covariance matrix as follows:

$$\begin{aligned} & XLX^H = X(D-S)X^H \\ & = \tfrac{1}{n} X(I - \tfrac{1}{n} e e^T) X^H \\ & = \tfrac{1}{n} X X^H - \tfrac{1}{n^2} (Xe)(Xe)^H \\ & = \tfrac{1}{n} \textstyle\sum_i x_i x_i^H - \tfrac{1}{n^2} (n\bar{x})(n\bar{x})^H \\ & = \tfrac{1}{n} \textstyle\sum_i (x_i - \bar{x})(x_i - \bar{x})^H + \tfrac{1}{n} \sum_i x_i \bar{x}^H + \tfrac{1}{n} \sum_i \bar{x} x_i^H - \tfrac{1}{n} \sum_i \bar{x}\bar{x}^H - \bar{x}\bar{x}^H \\ & = E(x_i - \bar{x})(x_i - \bar{x})^H + 2\bar{x}\bar{x}^H - 2\bar{x}\bar{x}^H \\ & = E(x_i - \bar{x})(x_i - \bar{x})^H \end{aligned} \tag{22}$$

where $\bar{x} = \frac{1}{n} \sum_i x_i$ is the mean of all the samples, and $E(x_i - \bar{x})(x_i - \bar{x})^H$ is the covariance matrix similar with S_t. If we take ε to be infinity and select the eigenvectors according to the largest eigenvalues of XLX^H, the feature vectors are projected along the directions of maximal variance. Hence, the global structure is preserved similar with GPCA.

3.4. **Connections with GFDA.** The projection matrix GFDA is the solution of the generalized equation $S_b W = \lambda S_w W$ where S_b and S_w are defined in section 2.2. Suppose there are a total of n classes, each class of m samples,let x_i^j denotes the sample j of class i, and $\bar{x}_i$ denotes the mean vector of class i, then

$$\begin{aligned} & S_w = \textstyle\sum_{i=1}^n (\sum_{j=1}^m (x_i^j - \bar{x}_i)(x_i^j - \bar{x}_i)^H) \\ & = \textstyle\sum_{i=1}^n (\sum_{j=1}^m (x_i^j (x_i^j)^H - \bar{x}_i (x_i^j)^H - x_i^j (\bar{x}_i)^H + \bar{x}_i (\bar{x}_i)^H)) \\ & = \textstyle\sum_{i=1}^n (\sum_{j=1}^m x_i^j (x_i^j)^H - m\bar{x}_i (\bar{x}_i)^H) \\ & = \textstyle\sum_{i=1}^n (X_i X_i^H - \frac{1}{m} (x_i^1 + \cdots + x_i^m)(x_i^1 + \cdots + x_i^m)^H) \\ & = \textstyle\sum_{i=1}^n (X_i I X_i^H - \frac{1}{m} X_i (e_i e_i^H) X_i^H) \\ & = \textstyle\sum_{i=1}^n X_i L_i X_i^H \end{aligned} \tag{23}$$

where $X_i L_i X_i^H$ is the covariance matrix, $X_i = [x_i^1, x_i^2, \cdots, x_i^m]$ is the $d \times m$ sample matrix of class i. $L_i = I - \frac{1}{m} e_i e_i^H$ is a m-dimensional vector. Let X represents the total sample

matrix, i.e. $X = [X_1, X_2, \cdots, X_n]$, and

$$w_{ij} = \begin{cases} \frac{1}{m^2} & if\ x_i\ and\ x_j\ both\ belong\ to\ the\ same\ class \\ 0 & otherwise \end{cases} \tag{24}$$

$$L = I - W \tag{25}$$

Then, we have

$$S_w = XLX^H \tag{26}$$

w_{ij} represents the similarity between x_i and x_j, so W can be taken as the weight matrix of a graph with data points as its nodes. The matrix L is thus regarded as graph laplcaian. Similarly, the matrix S_b can be computed as follows:

$$\begin{aligned} & S_b = \textstyle\sum_{i=1}^{n} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^H \\ & = (\textstyle\sum_{i=1}^{n} \bar{x}_i \bar{x}_i^H) - \bar{x}(\textstyle\sum_{i=1}^{n} \bar{x}_i^H) - (\textstyle\sum_{i=1}^{n} \bar{x}_i)\bar{x}^H + \textstyle\sum_{i=1}^{n} \bar{x}\bar{x}^H \\ & = (\textstyle\sum_{i=1}^{n} \frac{1}{m^2}(x_i^1 + \cdots + x_i^m)(x_i^1 + \cdots + x_i^m)^H) - 2n\bar{x}\bar{x}^H + n\bar{x}\bar{x}^H \\ & = \textstyle\sum_{i=1}^{n} \sum_{j,k=1}^{m} x_i^j x_i^k - n\bar{x}\bar{x}^H \\ & = XWX^H - n\bar{x}\bar{x}^H \\ & = XWX^H - X(\tfrac{1}{n} ee^H)X^H \\ & = X(W - \tfrac{1}{n} ee^H)X^H \\ & = X(W - I + I - \tfrac{1}{n} ee^H)X^H \\ & = -XLX^H + X(I - \tfrac{1}{n} ee^H)X^H \\ & = -XLX^H + C \end{aligned} \tag{27}$$

where $e = (1, 1, \cdots, 1)^H$ is a $n-$dimensional vector and $C = X(I - \frac{1}{n} ee^H)X^H$ is the covariance matrix. The generalized eigenequation $S_b W = \lambda S_w W$ of GFDA can be computed as follows:

$$\begin{aligned} & S_b W = \lambda S_w W \\ \Rightarrow\ & (C - XLX^H)W = \lambda XLX^H W \\ \Rightarrow\ & CW = (1 + \lambda)XLX^H W \\ \Rightarrow\ & XLX^H W = \tfrac{1}{1+\lambda} CW \end{aligned} \tag{28}$$

Then the projection matrix W can be obtained by solving the following equation:

$$XLX^H W = \lambda CW \tag{29}$$

If the mean value of the sample set is zero, the convariance matrix is XX^H which is exactly the matrix XDX^H in GLPP with the weight matrix defined in equation(24). It is known that GFDA aims to preserve discriminative information and global geometrical structure.

4. **Experiments.**

4.1. **Feature extraction and feature normalization.**

Before complex fusion, we should solve two problems: feature extraction and feature normalization. For the first problem, face and palm are easy to capture and register comparing with other biometrics. This paper selects them as two distinct biometric characteristics to test our algorithm. The experiments were performed on ORL face database and PolyU palm database.ORL face database includes 40 people, 10 different image with pose and expression variation per person. PolyU multispectral palm images were collected from 250 volunteers, including 195 males and 55 females, the samples were collected in two separate sessions. In each session, the subject was asked to provide 6 images for each palm. In order to fuse two feature sets, the number of samples should be coordinated. Our solution is to select 40

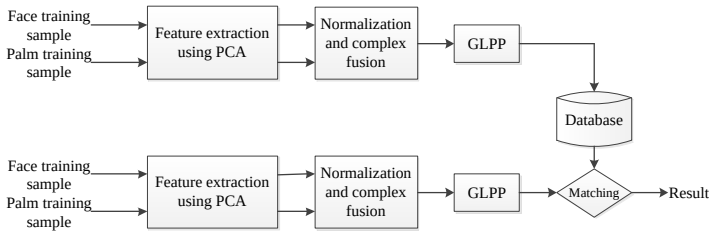

FIGURE 1. The framework of the proposed algorithm

classes in which three samples per class were selected as the training sets, three samples for testing.

Without loss of generality, PCA is taken as the unimodal feature extractor for face and palm. A sample image (a face image or a palm image) is pulled into a long vector as the input of PCA by progressive scanning. However, the size of two biometric images is different. So, the output of PCA for two biometrics isn't uniform. We can solve this problem by controlling the number of the eigenvalues of the covariance matrix of PCA. In other words, we only need to select the same number of the eigenvalues in descending order. Then the dimension of the projection matrix of two biometrics is same because the number of the eigenvalues composing the projection matrix is determined by the eigenvalues. In following fusion experiment, we select 80 dimension which doesn't bring a large amount of calculation and preserves the potential discriminant information for fusion. As a part of PCA, the above operation can be taken as feature normalization before fusion.

Besides consistent dimension, feature normalization includes eliminating the differences in the order of magnitude and the distribution between two distinct feature sets. We use the z-score model [5] to normalize two feature sets before fusion. Suppose $A = [a_1, a_2, \cdots, a_k, \cdots, a_m]$ be a feature matrix where a_k is the kth sample. Let A_k be the kth row of A, compute

$$D_k = \frac{A_k - \bar{A}_k}{\sigma_k} \tag{30}$$

where $\bar{A}_k$ is the mean value of A_k, σ_k is the standard deviation. Then A_k is normalized as following:

$$X_k = \frac{D_k - D_{min}}{D_{max}} \tag{31}$$

where D_{min} and D_{max} are the minimum value and the maximum value respectively. For all the rows of A, the normalization can be completed by implementing the same operations. Then the flow of the above procedure is illustrated in fig. 1.

4.2. **Experimental results and analysis.** Our goal is to compare our algorithm with unimodal biometric characteristics (face and palm), and another fusion approaches: series rule, weighted sum rule, sum rule(as a speical case of weighted sum rule), GPCA, GFDA, GKPCA and GKFDA. Table 1 shows the comparison of the above algorithms. DET curves are demonstrated in fig. 2. It can be clearly seen that our algorithm not only exceeds the performance of unimodal biometrics, but also outperforms the other seven fusion approaches. Compared with serial rule, sum rule and weighted sum rule, GLPP can also be considered as a feature selector to remove more redundancy of features before matching.

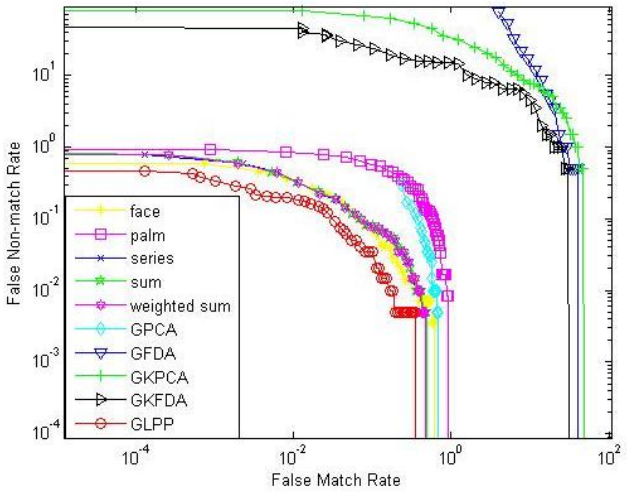

FIGURE 2. DET curve of the proposed algorithm

While GPCA attempts to preserve the global structure of the fusion space,and GFDA attempts to preserve the discriminating information; GLPP aims to preserve the intrinsic geometry of the data and local structure. In many real-world classification problems, the local manifold structure is more important than the global Euclidean structure, especially when nearest-neighbor like classifiers are used for classification [1]. Among subspace learning algorithm for face recognition, KPCA and KFDA are used to extract the nonlinear feature to resist the unwanted variations such as lighting, face expression and pose. However, they are computationally expensive. GLPP takes advantage of the optimal linear approximations to the eigenfunctions of the Laplace Beltrami operator to find the nonlinear manifold structures. Besides, GLPP encodes more discriminating information in the low-dimensional subspace by preserving adjacency relation. So GLPP is superior to GPCA, GFDA, GKPCA and GKFDA.

TABLE 1. EER comparison of different algorithms

Algorithm	face	palm	series	sum	weighted sum
EER (%)	8.2	29	8.5	8.3	8.4
Algorithm	GPCA	GFDA	GKPCA	GKFDA	GLPP
EER (%)	19.9	11.6	8.4	6.5	5.3

5. **Conclusion.** Feature fusion is the key problem of multimodal biometric recognition. Comparing with the traditional feature fusion rules, complex field fusion avoids the calculation consumption of the series rule, and the weight determination of the weighted sum rule. Generalized locality preserving projection(GLPP) is proposed to extend LPP into the complex field to perform multimodal feature fusion and classification. Comparing with GPCA and GFDA, GLPP preserves local information and obtains a locality preserving subspace that best detects the essential sample manifold structure. Simultaneously, GLPP avoids calculation consumption of GKPCA and GFDA. Face and palm are used as two distinct biometric modals to test our algorithm. Experimental results show that

the proposed algorithm achieves much better performance than two unimodal biometrics (face recognition and palm recognition) and other conventional multimodal biometric algorithms(GPCA, GFDA, GKPCA and GKFDA).

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