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Modeling and Optimization of The Four-Coil Wireless Power Transfer System

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Abstract. *The four-coil type Wireless power transfer (WPT) system, which is based on magnetic resonance coupling, can be widely used in the mid-range WPT applications. As a power converter, the analytic model, which consist of efficiency and output power calculation, is the most important theoretical basis for the optimization of WPT system.In this paper, a analytic model of the four-coil type system is built and verified. It overcome the drawbacks of present models, which are too complex to used in general design of WPT system. With the formulas of the model proposed, the efficiency optimization can be achieved easily through coupling coefficient and load resistance selection at particular parameters condition. A WPT prototype based on FEM simulation is built for verifying the model proposed. The transfer efficiency, output power and load feature are matched well between the calculation and PSPICE simulation. The model proposed is proved to have high accuracy. In the prototype, the peak transfer efficiency reaches 95.33% at rated parameters. Meanwhile, the transfer efficiency is improved significantly with the optimized R_L when the transfer distance variates.*
Keywords: Four-coil, wireless power transfer (WPT), Resonance, Efficiency,Optimization

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1. **Introduction.** Wireless power transfer (WPT) based on magnetic resonance coupling is a novel approach of transmitting electric power [1–5]. It can overcome the drawbacks of traditional cable power transmission, such as line aging, fixed position, and mess. With the resonators, which are consist of coils compensated by capacitors, it can transmit power wirelessly and efficiently. According to the number of coils, the WPT system can be classified in to tow-coils and four-coil type. It has been proved in many literatures that the four-coil type can better satisfy the requirements of mid-range WPT applications.

Due to the energy transmission principle of WPT, it requires at least two coils. As shown in Fig.1, the basic two-coils type based on series-series(SS) compensation can realize the highest transfer efficiency theoretically. However, in the two-coils WPT system, with the increase of transfer distance, the coupling coefficient between coils will decrease rapidly,and the transfer efficiency also reduces greatly [6–9].

Literature [10] proposed a maximum energy efficiency tracking method for the two-coil WPT system. It shows that the efficiency model is very complex even for the two-coil type system.The efficiency of two-coil WPT system has been built in detail [11, 12]. However, it suits the two-coil but not the four-coil WPT system.A quantitative analysis of transfer

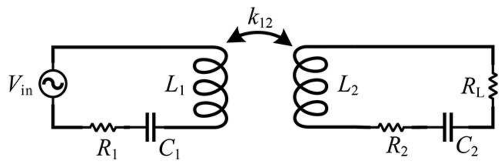

FIGURE 1. Basic two-coil WPT system

efficiency in four-coil WPT system was accomplished in [13]. It provides a new way to optimal the transfer efficiency. However, the analytic equations in the model are not based on quality factor design of coupling coils, and the equations are too complex to apply in the actual design of four-coil WPT system. Using circuit simulation software is a potential ideal method for actual design of the four-coil WPT system due to the development of computer technology.However, it is very difficult to get the optimum parameters for general design.

In consideration of the drawbacks in present models, this paper focuses on the analytic model and optimization of four-coil WPT system.Firstly, the basic transfer principle,which includes efficiency and output power formula, based on impedance feature is analyzed.And then, with the differential calculation of the formulas, optimization parameter equations of coupling coefficients at particular quality factor condition are proposed. A prototype design based on the model is built for verification. The transfer efficiency, output power and load feature are matched well between the calculation and simulation. Therefore, the model proposed is proved to have high accuracy. In the prototype, the peak transfer efficiency reaches 95.33% at rated parameters. When the transfer distance variates, the transfer efficiency is improved significantly with the optimized R_L.

2. **Basic Principle.**

2.1. **Impedance Characteristic.** As shown in Fig.2, the four-coil WPT system contains four coils, driving coil L_1, transmitting coil L_2, receiving coil L_3, and loading coil L_4. The power transfer depends on the magnetic coupling between coils.The coupling coefficient between L_1 and L_2 is defined as k_{12}. Similarly, there are also k_{23}, k_{34}, k_{14}, k_{24} and k_{13}, respectively. Due the variation characteristic of magnetic coupling, the k_{13}, k_{24} and k_{14}, which are far less than k_{12}, k_{23} and k_{34}, can be ignored for simplifying the analysis. The capacitors,i.e. C_1, C_2, C_3 and C_4, are connected to the coils in series for resonance coupling compensation. The R_1,R_2,R_3 and R_4 means the ESR (Equivalent Series Resistance) of each coil and its compensation capacitor, respectively. The ESR is depend on the design factor of coil, such as coil size, conductor, capacitor material, and so on. The R_L means load resister which consumes the output power. Transfer efficiency is defined as the factor between output and input power.

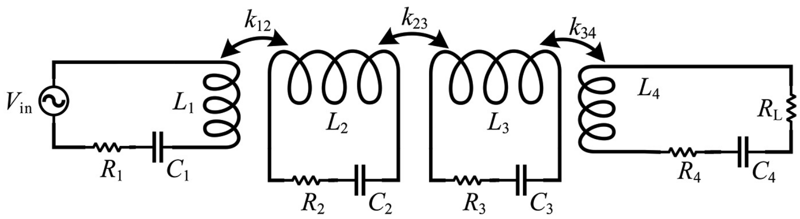

FIGURE 2. Equivalent circuit of four-coil WPT system

The input power depends on the input source and the input impedance. Similarly, the output power depends on the output voltage/current and the load resister.Therefore, they can easily get by impedance analysis.Defining Z_{1lop} to Z_{4lop} as the loop impedance of each compensated coil. According to the principle of magnetic resonance coupling WPT system, the resonance frequency of every resonator is the same. Define the resonant angular frequency as ω_o, and the coil inductance and its compensation capacitor meet formula:

$$j\omega_o L_n + \frac{1}{j\omega_o C_n} = 0 \tag{1}$$

Where n equals 1 to 4. Therefore, the loop impedance can be calculated by

$$Z_{4lop} = j\omega_o (n - 1/n) L_4 + R_4 + R_L \tag{2}$$

$$Z_{3lop} = j\omega_o (n - 1/n) L_3 + R_3 + Z_{r3} \tag{3}$$

$$Z_{2lop} = j\omega_o (n - 1/n) L_2 + R_2 + Z_{r2} \tag{4}$$

$$Z_{1lop} = j\omega_o (n - 1/n) L_1 + R_1 + Z_{r1} \tag{5}$$

$$Z_{r3} = (k_{34} n\omega_o)^2 L_3 L_4 / Z_{4lop} \tag{6}$$

$$Z_{r2} = (k_{23} n\omega_o)^2 L_2 L_3 / Z_{3lop} \tag{7}$$

$$Z_{r1} = (k_{12} n\omega_o)^2 L_1 L_2 / Z_{2lop} \tag{8}$$

$$n = \omega / \omega_o \tag{9}$$

Where Z_{r1} to Z_{r4} are the reflection impedances [14]. ω means the operation angular frequency, and when it equals to ω_o, i.e. $n = 1$ the WPT system operates at resonance, which is normally the expected operation condition. With the equations above, the efficiency or other operation feature of system can be easily deduced.

2.2. **Efficiency and Output Power.** As mentioned above, when the system is operate at resonance condition, i.e. $n = 1$, the input power can be calculated by

$$P_{in} = \frac{U_{in}^2}{R_{lop1}} \tag{10}$$

$$R_{lop1} = R_1 + Z_{r1} \tag{11}$$

Meanwhile, the efficiency of each coil loop can be easily calculated by

$$\eta_{lop4} = R_L / (R_L + R_4) \tag{12}$$

$$\eta_{lop3} = Z_{r3} / (Z_{r3} + R_3) \tag{13}$$

$$\eta_{lop2} = Z_{r2} / (Z_{r2} + R_2) \tag{14}$$

$$\eta_{lop1} = Z_{r1} / (Z_{r1} + R_1) \tag{15}$$

$$\eta_{sys} = \eta_{lop1}\eta_{lop2}\eta_{lop3}\eta_{lop4} \tag{16}$$

Where η_{lop1}, η_{lop2}, η_{lop3} and η_{lop4} are the loop efficiency of L_1, L_2, L_3 and L_4, respectively. η_{sys} is the transfer efficiency of WPT system. It shows that the efficiency formulas of the four-coil WPT system is very concise. And then, the output power can be get through efficiency and input power, i.e.

$$P_{out} = P_{in} / \eta_{sys} \tag{17}$$

Now, the complete model of four-coil WPT system is built. In order to get the optimization parameters of system, which realize maximum transfer efficiency or output power, the analytic model above is not enough. Therefore, the optimization model will be deduced as follow sections.

2.3. **Optimization of Parameters.** The transfer efficiency is decided by loop efficiency of each coil, which is listed above. In consideration of the variable parameters which will affect the transfer efficiency, the WPT system can be optimized by parameters selection. In the actual design of magnetic coupling based WPT system, load resistor, quality factor of coils and their coupling coefficients decide the transfer feature, which includes efficiency and power flows. At normally condition, higher quality factor, i.e. Q value of coil is preferred. Therefore, the Q value is limited by cost factor, and the optimization of parameters equates with the selection of coupling coefficients , which are decided by the relative position of coils, and load resistor.

In order to get the optimal values of coupling coefficients, i.e. k_{12}, k_{23} and k_{34}, performing analytic derivatives on (16). Take derivative of η_{SYS} with respect to the k_{12}, i.e.

$$\frac{d\eta_{sys}}{dk_{12}} = \frac{2Q_1{Q_2}^2{Q_3}^2Q_4k_{12}k_{23}^2{k_{34}}^2}{\left(Q_1Q_2Q_3Q_4{k_{12}}^2{k_{34}}^2 + \alpha + \gamma + \sigma Q_4 - 1\right)^2} \tag{18}$$

$$\alpha = Q_1Q_2{k_{12}}^2 + 1,\ \gamma = Q_2Q_3{k_{23}}^2 + 1,\ \sigma = Q_3{k_{34}}^2 \tag{19}$$

$$Q_n = \frac{\omega_o L_n}{R_n},\ Q_L = \frac{\omega_o L_4}{R_L} \tag{20}$$

Where Q_1 to Q_4 are the quality factors of coils, and the Q_L is defined as load quality factor.Therefore, the derivative is always great than 0, which is also mentioned in [13]. That means, transfer efficiency is alway increase with k_{12}. Meanwhile, the Z_{r1} increases with k_{12}, and the output power decreases. Therefore, the k_{12} selection depend on the requirement of transfer efficiency and output power, and a higher k_{12} is preferred.

Take derivative of η_{SYS} with respect to k_{23}, k_{34}, Q_L, and deducing their root, i.e.

$$\frac{d\eta_{sys}}{dk_{23}} = 0,\ \frac{d\eta_{sys}}{dk_{34}} = 0,\ \frac{d\eta_{sys}}{dQ_L} = 0 \tag{21}$$

Solving (21), we can get the optimization equations of k_{23}, k_{23}, Q_L, i.e. k_{23optm}, k_{34optm}, Q_{Loptm} as

$$k_{23optm} = \frac{\alpha^{1/4}(\sigma/\beta + 1)^{1/2}}{(Q_2Q_3)^{1/2}} \tag{22}$$

$$k_{34optm} = \frac{\gamma^{1/4}\,(\alpha{+}\gamma{-}1)^{1/4}}{\alpha^{1/4}(Q_3/\beta)^{1/2}} \tag{23}$$

$$Q_{Loptm} = \frac{Q_4\gamma^{1/2}\,(\alpha + \gamma - 1)^{1/2}}{(\gamma + \sigma Q_4)^{1/2}[(\sigma Q_4 + 1)\,\alpha + \gamma - 1]^{1/2}} \tag{24}$$

$$\beta = \frac{1}{Q_4} + \frac{1}{Q_L} \tag{25}$$

Compared with present models, the optimization equations is based on quality factors, and it is suitable for the actual design of four-coil WPT system. With the equations above, the best values of parameters for efficiency optimization can be easily get at particular design conditions. A prototype is built at following sections for verifying the analytic model proposed.

3. **Four-Coil WPT Prototype Design and Optimization.**

3.1. **Parameters of Prototype.** As mentioned above, the transfer efficiency have direct correlations with coupling coefficient and load resistor. In order to simplify the design and analysis, the prototype is designed symmetry, i.e. $L_1 = L_4$, $L_2 = L_3$, $Q_1 = Q_4$, $Q_2 = Q_3$. The four-coil WPT prototype is illustrated in Fig.3 with the parameters in Tab.1. The coupling coils are all 5 turns with 1mm diameter copper wire. The radius of transmitting and receiving coil is 100mm, and that of driving and load coil is 50mm. The parameters in TAB.1 are got by FEM(Finite Element Method) simulation.

The transfer distance is rated at 100mm, and the rated resistor is 35 Ω. As mentioned above, a higher k_{12} is preferred, but it also affects the output power. Therefore, k_{12} should be adjusted due to the requirement of output power. In this paper, k_{12} is fixed constant.

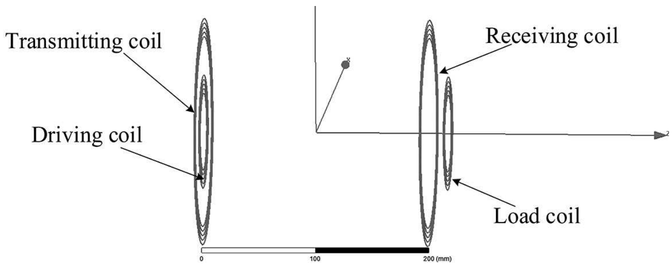

FIGURE 3. FEM model for the prototype

TABLE 1. System Parameters

Parameter	Value
Resonance Frequency	$f_o = 6.78$MHz
Coil Inductance	$L_1 = L_4 = 3.1727\mu$H, $L_2 = L_3 = 9.1856\mu$H
Coupling Coefficients	$k_{12} = 0.1973$, $k_{23} = 0.1084$ at rated transfer distance
ESR	$R_1 = R_4 = 318.85$mΩ, $R_2 = R_3 = 693.46$mΩ
Rated Load Resistance	$R_0 = 35\Omega$
Quality Factor of Coils	$Q_1 = Q_4 = 423.89$, $Q_2 = Q_3 = 564.28$
Compensation Capacitor	$C_1 = C_4 = 0.174$nF, $C_2 = C_3 = 0.06$nF

3.2. **Optimization of k_{34} at Rated Conditions.** In order to get the highest transfer efficiency at rated conditions, the optimal value of k_{34} should be calculated based on the analytic model above. The optimal value of k_{34} equals **0.18315** by substituting the parameters of Tab.1 into equation (23), and the highest efficiency reaches **95.334%**. According to the results of FEM simulations as in Fig.4, the optimal k_{34} corresponds to 17mm distance between receiving and load coils. In the PSPICE simulation, the transfer efficiency is **95.334%**, which matches the calculated values exactly. Therefore, the efficiency equation,i.e. (16) is proved accurate.

In order to verify the optimization equation of k_{34}, i.e. (23). Simulating the variation of transfer efficiency with k_{34} at rated conditions, as in Fig.5. It shows that the transfer efficiency reaches peak value at $k_{34} \approx 0.18$. Therefore, the optimization model is proved accurate.

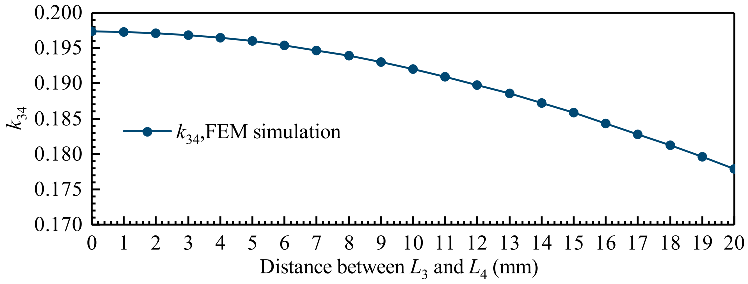

FIGURE 4. The variation of k_{34} with the distance between L_3 and L_4

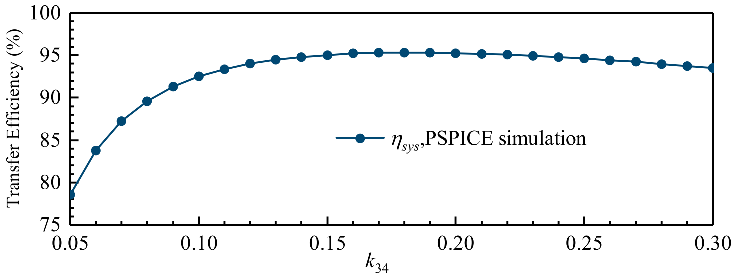

FIGURE 5. The variation of transfer efficiency with k_{34}

3.3. **Optimization of R_L With the Variation of Transfer Distance.** After the optimization above, the four-coil WPT prototype can achieve the highest efficiency at rated transfer distance. However, in the actual WPT system, the transfer distance often variates, which causes k_{23} variation, and the best operation point will also shift. According to the analytic model above, the coupling coefficient is fixed constant after system design. Therefore, in order to realize better efficiency, the R_L should be changed with the variation of transfer distance.

In the prototype, the transfer distance variates from 50 to 200mm at different operation conditions. As in Fig.6, k_{23} variates from to , which is got by FEM simulation. The transfer efficiency decreases shapely with the increase of transfer distance as in Fig.7. To retard the decrease, variating the R_L based on equation (24). As shown in Fig.7, the transfer efficiency with R_L optimized is always higher than that of rated load resistance, especially when the transfer distance greater than the rated value.

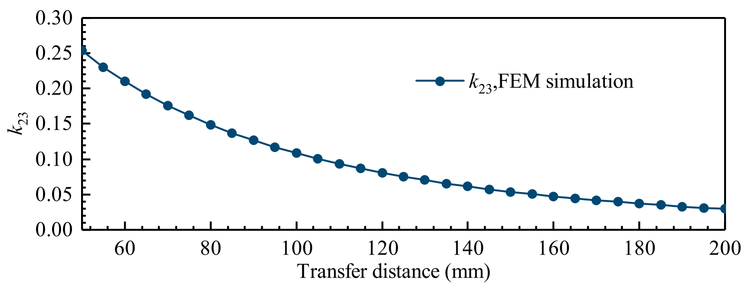

FIGURE 6. The variation of k_{34} with transfer distance

With the optimization of R_L, the transfer efficiency is improved significantly. Therefore, the analytic model of four-coil WPT system proposed can be used in the controller. For

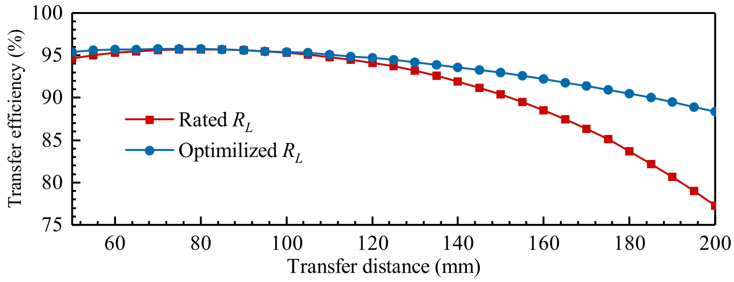

Figure 7. The comparison of η_{SYS} between rated and optimized R_L

instance, applying a power converter, which can changing load resistance equivalently, to the receiver.

3.4. **Trade off Between Transfer Efficiency and Output Power.** The output of four-coil WPT system, which is excited by voltage source, behaves as current source. Therefore, the output power increase with the load resistance. As in Fig.8, the output power, at 48 volts amplitude input, with rated R_L and optimized R_L presents different features. The best efficiency and output power requirement may not match well. There is trade off between transfer efficiency and output power. To realize stable output power, the R_L should be increased when the transfer distance increase. On the contrary, R_L should be decreased when the transfer distance decrease. To get best efficiency, the R_L should be set according to the optimized value.

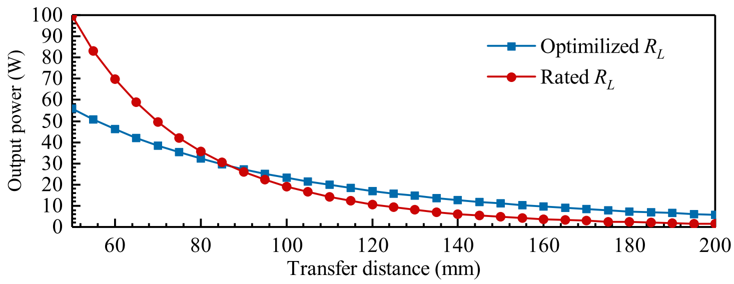

Figure 8. The comparison of P_{out} between rated and optimized R_L

4. **Conclusions.** In this paper, the analytic model and optimization of four-coil WPT system is proposed based on impedance analysis. And then, with the differential calculation of the formulas, optimization parameter equations of coupling coefficients and load resistance at particular quality factor condition are proposed. A prototype design based on FEM simulation is built. The transfer efficiency, output power and load feature are matched well between the calculation and PSPCIE simulation. The analytic model proposed is proved to have high accuracy. There are optimal values of k_{23}, k_{34}, R_L to get best transfer efficiency. The four-coil WPT system can be easily optimized with the equations proposed. The peak transfer efficiency of prototype reaches 95.33% at rated parameters with the optimization. When the transfer distance variates, the transfer efficiency is improved significantly with the optimized R_L.

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