

## Finding Most Likely Sliding Surfaces Using PSO

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**Abstract**—PSOslope is a Windows application program with a GUI interface written in C# using the evolutionary algorithm of Particle Swarm Optimization (PSO). It was designed and developed to compute the Factor of Safety (FS) of a soil slope. This paper presents a new version of the program called PSOslope 2, which contains a number of improvements over the previous version, including the support for the Bishop's Simplified Method (BSM) in addition to the existing Ordinary Method of Slices (OMS). The new program also used more stringent criteria for assessing acceptable sliding surfaces than the old version did. Finally, PSOslope 2 was used to compute the FS of a simple vertical cut in a homogeneous soil. The results showed that PSOslope 2 was able to discover a more likely sliding surface (smooth but non-planar) with a lower value of FS than the theoretical prediction (based on the assumption of planar failure). This demonstrated that the PSO could be used to efficiently optimize the problems involving slope stability that includes natural slopes and excavations. It also showed that the PSO was a promising approach and provided practical solutions for the search of most likely sliding surfaces.

**Keywords**—landslides, C++, slopes, PSO

### I. INTRODUCTION

Taipei is located in a basin surrounded by mountains on the north, east, and south, and drained by the Tamsui River flowing to the west. To the north, igneous rock hardened from lava erupted from the dormant Datun volcanoes covers most of the area. To the east and south, sedimentary rocks compacted from sand, clay, and mud are the most common outcrops as well as bedrocks underneath the loose surface deposits of soil and alluvium. The weather is sub-tropical with hot and humid summers, and the average annual rainfall is 2932 mm based on statistics from 1947 to 2007. Although different rock types are found at different locations, landslides are a common problem in the mountains around Taipei due to intense rainfalls brought by summer typhoons and plum rains. Over the last few decades, concern over the degradation of the environment has increased and new technology has been developed to make it possible to study the finer details of landslides. Technological advances such as the high resolution 3D terrestrial laser scanners (also called LiDAR, short for Light Detection and Ranging) and the GPS have been used extensively in the study of landslides and slope stability problems. An example of such

a study is the detailed monitoring and scanning of the large landslide site near the Houshanyue hiking trail (see Fig. 1) located in the Wenshan District of Taipei City [1]. The Houshanyue landslide is consisted of a number of smaller landslides with one of them shown in Fig. 2. It is a slope resting at the angle of repose, and the surface profile is approximately straight. Because of ground subsidence, scarps on both sides of the landslide are clearly visible. At the bottom of the scarp and underneath the surface soil, layers of weakly cemented rock can be observed. This entire area has been laser scanned and put in a digital format. However, the advances in the analysis of slopes have not paralleled the advances in laser scanning. More often than not, the method to locate the critical sliding surface of a slope is still the same as decades ago. To keep up with the advances in technology, this paper investigates and presents the second iteration of the program that uses the evolutionary algorithm of Particle Swarm Optimization (PSO) to improve the determination of the most critical sliding surface of a slope. It is anticipated that after a few more iterations the PSO program will be sophisticated enough to analyze the massive amount of data generated by laser scanning and produce meaningful results.

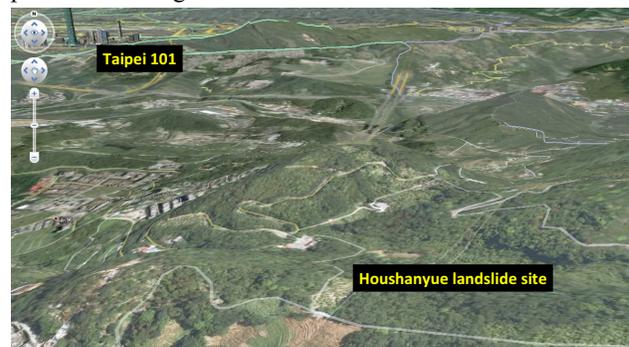


Figure 1. Location of the Houshanyue landslide site as seen in Google Earth. The Taipei 101 tower can be seen in the background.

### II. PREVIOUS WORK

PSO is an evolutionary algorithm developed by Eberhart and Kennedy [2-3] and has been applied to many diverse fields [4-6]. It is considered to be a straightforward, powerful, and flexible method to solve optimization problems using a simple iterative equation:

$$v = v + 2 * r1 * (pbest - x) + 2 * r2 * (gbest - x) \quad (1)$$

where  $v$  and  $x$  are the velocity and the position of the individual agent respectively, and  $r1$  and  $r2$  are random numbers. Coincidentally, the analysis of slope stability usually involves the search of the smallest Factor of Safety (FS) for a given slope using the method of slices (see Fig. 3) under the principal of limit equilibrium. The analysis constitutes an optimization problem, which is classically considered to be computationally intensive and hence relies on numerical solutions. Consequently, it seems very logical to apply the PSO to the computation of FS. For a typical soil slope shown in Fig. 3, the FS can be formulated as:

$$FS = \frac{\sum [c_i l_i + W_i \cos \alpha_i \tan \phi]}{\sum W_i \sin \alpha_i} \quad (2)$$

where  $c$  = cohesion,  $\phi$  = friction angle,  $l$  = the length of the slice base,  $\alpha$  = the angle of the slice base, and  $W$  = the soil weight of the slice. Previously, a stand-alone Windows application called PSOslope was developed to evaluate (2) with a GUI interface [7]. It has five groups of input boxes designed for the definition of the slope profile, the initial sliding surface, the soil properties, the number of agents and iterations, and the parameters used in the PSO formulation. The screenshot of PSOslope is shown in Fig. 4. It was implemented with C#.



Figure 2. One of the landslides at the Houshanyue site with scarps and rock layers shown in the picture.

### III. DEVELOPMENT OF NEW PROGRAM

PSOslope was a proof-of-concept project aimed to provide a Windows application that could be used to study the influence of various slope parameters on the final shape of the sliding surfaces. It was a big step forward towards the intuitive and graphical analysis of slope stability. For example, the line segments of various colors in Fig. 4 representing the sliding surfaces are analogous to birds and fishes in the swarm in search of food and other resources. They move rapidly on the computer screen to give the user an instant feedback of the computational process and a visual clue of the convergence behavior. In order to fully explore the solution space and the odd shapes of sliding surfaces, PSOslope computed with the least amount of restriction or constraints. While interesting results were observed, problems and computation dilemmas have also resulted. To

further improve on the results of computation and address the issue of kinematical admissibility, a new Windows application called PSOslope 2 has been developed. It is a complete rewrite of the original PSOslope from scratch using C++. A number of important functionalities have been incorporated into the new program as described in the following sections.

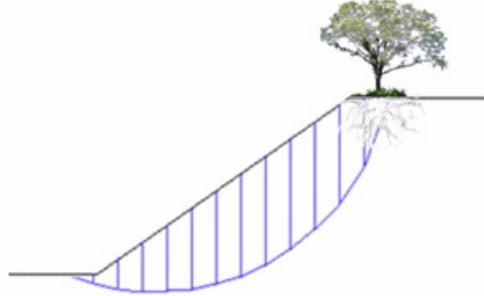


Figure 3. Illustration of the method of slices in the determination of the most likely sliding surface of a soil slope.

#### A. Addition of New Analysis Capability

The original PSOslope used the Ordinary Method of Slices (OMS) in the computation of FS as formulated in (2). PSOslope 2, on the other hand, used the Bishop's Simplified Method (BSM) in addition to the OMS. The screenshot of PSOslope 2 is shown in Fig. 5. Most of the input boxes of PSOslope2 are the same as those of the original PSOslope. A new drop down box was added to the lower right-hand corner of the screen to allow the option of choosing between OMS and BSM. Furthermore, three additional checkboxes were implemented to remove unreasonable sliding surfaces: (1) apply angle restrictions, (2) remove negative (surfaces), and (3) remove unreasonable arc(s). The checkboxes are independent to one another and can be checked in any combination. When the boxes are checked, PSOslope 2 will generate smoother concave-up sliding surfaces.

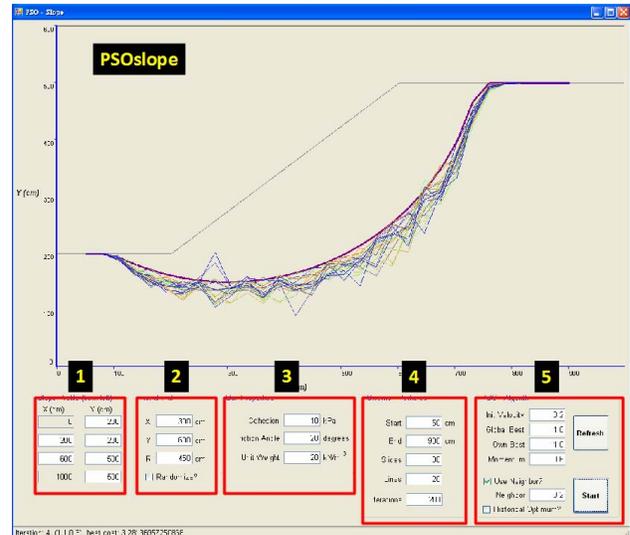


Figure 4. A screenshot of PSOslope developed using C#.

### B. Back to the De Facto Programming Language

As mentioned earlier, PSOslope 2 is a complete rewrite of the original PSOslope. The original programming language of C# was replaced in favor of the "de facto" programming language of C++. Arguably the most widely used programming language, C++ has the advantages of wide user base, easy code maintenance, potential code reuse, and extensibility.

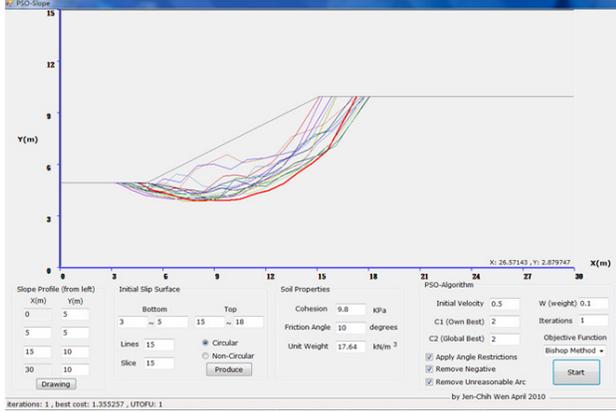


Figure 5. A screenshot of PSOslope 2 developed using C++.

### C. Interface Designed to Fit a Wide Screen Layout

The user interface of PSOslope 2 remains mostly the same as the original PSOslope. The aspect ratio changes slightly to better accommodate widescreen monitors that are the standard these days. This also solves the display problem of PSOslope, which required more vertical pixels than were available on the standard laptop screen (usually 1024 by 768 pixels, or 786432 pixels total).

## IV. EXAMPLE ANALYSIS

As a verification of PSOslope 2, a test case was chosen from the literature [8]. It is a simple vertical cut in a homogeneous soil of a zero friction angle and can be considered as a special case of a slope with 90° slope angle (as shown in Fig. 6). If the sliding surface is assumed to be planar (a straight line in Fig. 6), the FS of the slope can be formulated as:

$$FS = \frac{cH / \cos \beta}{\frac{1}{2} \gamma H^2 \tan \beta \cos \beta} = \frac{4c}{\gamma H \sin 2\beta} \quad (3)$$

where  $c$  = cohesion,  $\gamma$  = the unit weight of the soil,  $H$  = the height of the cut,  $\beta$  = the angle of the sliding surface. The assumption of planar sliding surface is necessary in order to derive the formulation shown in (3). It should be understood that the sliding surfaces could be non-planar, cylindrical, or arbitrarily curved. For such surfaces, the theoretical solutions do not exist. In addition, there is no guarantee that planar sliding surfaces have the smallest values of FS compared to other types of sliding surfaces.

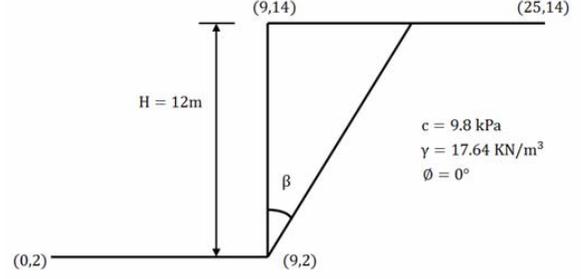


Figure 6. A simple vertical cut in a homogeneous soil with no friction angles.

### A. Theoretical Solution

Differentiate (3) with respect to  $\beta$  and set the resulting equation to zero gives the extreme value of FS:

$$\frac{d(FS)}{d\beta} = \frac{4c}{\gamma H} \times \frac{-2 \cos 2\beta}{(\sin 2\beta)^2} = 0 \quad (4)$$

Obviously, this is equivalent to:

$$\cos 2\beta = 0 \quad (5)$$

Equation (5) being zero indicates  $\beta = 45^\circ$ . When this occurs, the FS defined by (3) will have the extreme value, which happens to be the minimum value of FS:

$$FS = \frac{4c}{\gamma H \sin 2\beta} = \frac{4 \times 9.8}{17.64 \times 12 \times \sin 90^\circ} = 0.1852 \quad (6)$$

In other words, FS has a minimum value of 0.1852 when the sliding surface of the vertical cut is assumed to be planar. The critical sliding angle under this assumption is a constant value of 45°.

### B. Solution by PSOslope 2

For comparison purposes, the vertical cut in Fig. 6 was also analyzed by PSOslope 2. Out of the three checkboxes, only the one indicating "remove negative (surfaces)" was checked. This will have the effect of discarding the sliding surfaces with negative values of FS, and will prevent the program from diverging to negative infinity. No other restrictions were applied to make the sliding surfaces smoother. The analysis results are shown in Fig. 7. It can be observed from Fig. 7 that the optimum answer converges to a smooth and almost straight line (red line) despite of large initial oscillations (lines of various colors). Although not a perfectly straight line, the final shape of the most likely sliding surface is very similar to the theoretical solution (planar failure assumed). The angle  $\beta$  is measured to be 56.25°, close to 45°. The FS is 0.1769, smaller than the 0.1852 predicted by the theory. However, it should also be noted that the theoretical value was based on the assumption of planar sliding surfaces. Lower values of FS could exist for non-planar surfaces as in this example.

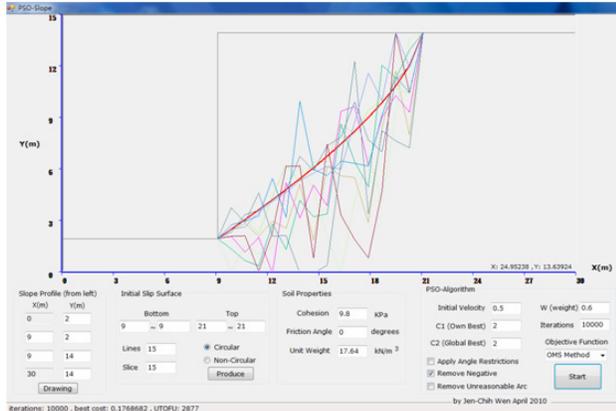


Figure 7. The most likely sliding surface (red line) discovered by the PSO using PSOSlope 2.

## V. SUMMARY AND CONCLUSIONS

Landslides are a common problem found all around the world. However, the advances in analysis method have not paralleled the advances seen in today's technologies such as the terrestrial laser scanning for the precise documentation and 3D modeling of sites and slopes. In order to improve the search of the most critical sliding surfaces in soil slopes, this paper presented a revised and re-written version of the slope stability analysis program based on the evolutionary algorithm of PSO. The newly implemented version, PSOSlope 2, contains a number of improvements, most notably the addition of BSM and the inclusion of algorithm functionalities to guarantee the convergence of FS and the appearance of concave-up sliding surfaces. As a test, PSOSlope 2 was used to compute the FS of a vertical slope in a homogeneous soil. It was found that the resulting most critical sliding surface discovered by the PSO algorithm was fairly smooth (without using the smoothing algorithms) and close to the planar failure surface usually assumed in the literature. The FS of 0.1769 obtained by PSOSlope 2 was smaller than the theoretical value of 0.1852 (computed under

the assumption of planar failure), and was more desirable for the purpose of minimizing the value of FS (most likely sliding surface) and consequently demonstrated the applicability of PSOSlope 2 in the analysis of slope stability.

## ACKNOWLEDGMENT

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