Advanced intelligent Trajectory Tracking Design for Vehicle Systems Under the effects of Uncertain Disturbances

Nai Ren Guo
Department of Electrical Engineering,
Tung Fang Design University
Kaohsiung County, Taiwan
nrguo@mail.tf.edu.tw

Yung-Yue Chen
Department of Electrical Engineering,
National Yunlin University of Science and Technology
Yunlin, Taiwan
chenyuyu@yuntech.edu.tw

Tzong Jiy Tsai
Department of Electrical Engineering,
Tung Fang Design University
Kaohsiung County, Taiwan
jiy@mail.tf.edu.tw

***Abstract*—An intelligent control of the automated highway system for the guidance of four wheeled vehicles is proposed. The control objective is to find one control law that can automatically guide the new-generation smart vehicles when driving in the highway under the effects of uncertain disturbance as wind gust, etc. We successfully propose one control law for designing the intelligent control of four wheeled vehicles based on concepts of GPS, fuzzy control, and robust control.**

Keywords-*trajectory tracking****; lateral control; fuzzy control; robust control***

I. INTRODUCTION

Lateral control has been as an active research topic that has focused on four main factors that influence system performance: vehicle handling characteristics, preview distance, actuator capability, and controller design.[5-8] Basically, this problem can be viewed as that of determining an appropriate control law for commanding the vehicle steering angle. Two approaches can be considered for the lateral control problem, one is the vehicle guidance and control problems into an outer guidance loop and an inner control loop, the other is an integrated approach wherein the inner and outer loops are designed simultaneously. In [1-2], a novel method has introduced to solve the active steering of 2-wheel steering vehicles problem which allows robust unilateral decoupling of the yaw rate from the lateral dynamics has been presented. The sliding mode control has been applied to the problem of trajectory control [1-2], and is recently receiving increasing attention from researches on control with uncertainties.

II. THE VEHICLE MODEL

In this section, the dynamic, kinematic equations and the parameters of the vehicle model are described. For simplicity only the lateral and the yaw dynamics are considered (see Figure 1)[8]. This simplification captures the dynamics that would affect control design.

We write the dynamical equations for the vehicle in the co-rotational coordinates X , Y . The Euler-Newton equations for the planar rigid body of Figure 1 are

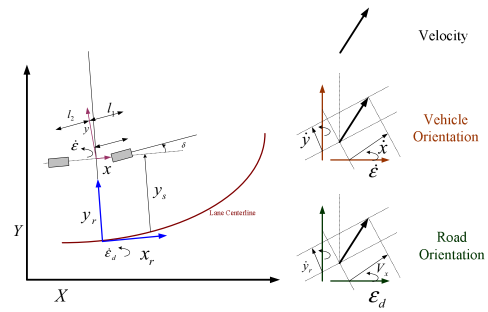

Figure 1. The description of the hicle model

$$\begin{aligned} m\ddot{y} &= -m\dot{\varepsilon}\dot{x} - C_{\alpha f}(\dot{y}-\dot{\varepsilon}l_2)/\dot{x} - C_{\alpha r}(\dot{y}-\dot{\varepsilon}l_1)/\dot{x} \\ &\quad + C_{\alpha f}\delta \\ I_z\ddot{\varepsilon} &= l_2 C_{\alpha f}(\dot{y}-\dot{\varepsilon}l_2)/\dot{x} - l_1 C_{\alpha r}(\dot{y}-\dot{\varepsilon}l_1)/\dot{x} \\ &\quad + l_1 C_{\alpha f}\delta \end{aligned} \tag{1}$$

where V_x and V_y are the components of the vehicle velocity along the x_r and y_r axes, respectively, as shown in Figure 1. $\dot{\varepsilon}$ is the yaw rate. m and I_z are the mass and the yaw moment of inertia respectively. l_1 and l_2 are distances of front and rear axle from the center of gravity. $C_{\alpha f}$ and $C_{\alpha r}$ are the front and rear cornering stiffness respectively. δ is the steering angle. The model described by equation (1) is independent of road reference. To describe the vehicle relative to the road, a road reference frame (x_r, y_r) is used. The rotation rate of this frame is defined as $\dot{\varepsilon}_d$. Note that $\dot{\varepsilon}_d = V_x\rho$, where ρ is the road curvature. The distance of the vehicle center gravity from the origin of frame (x_r, y_r) is defined as y_{cg} .

III. SYSTEM MODELING

In this paper, the equations of motion are written for the lateral error y_v . This is the lateral error at a distance d_v ahead of the center of gravity . However, from Figure 1, we see that

$$y_v = y_a + y_0 \tag{2}$$

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where y_a is the actual lateral error at d_v and y_0 is the offset because of the road curvature. Because y_0 can be computed from road geometry, without loss of generality, the problem of regulating y_v to zero is considered. The lateral error y_v can be expressed as

$$y_v = y_{cg} + d_v \varepsilon_r \quad (3)$$

Differentiating equation (2) twice, we get

$$\ddot{y}_v = \ddot{y}_{cg} + d_v \ddot{\varepsilon}_r + 2\dot{d}_v \dot{\varepsilon}_r + \ddot{d}_v \varepsilon_r \quad (4)$$

From Figure 1, assuming small ε_r, we get

$$\ddot{y}_{cg} = \dot{y}_c + \dot{x}\varepsilon_r$$

and

$$\ddot{y}_{cg} = \ddot{y}_c + \dot{\varepsilon}\dot{x} - V_x^2 \rho \quad (5)$$

where ρ is the curvature. Using (1), (4), (5) and assume $\ddot{d}_v = \ddot{\varepsilon}_d = 0$, we get the following model equations

$$\begin{aligned}\ddot{y}_v = &-\frac{(\phi_1+\phi_2)}{V_x}\dot{y}_v + \frac{(\dot{d}_v+\dot{x})}{V_x}(\phi_1+\phi_2)\varepsilon_r \\ &+\frac{\phi_1(d_v-l_1)+\phi_2(d_v+l_2)+2\dot{d}_v\dot{x}}{V_x}\dot{\varepsilon}_r \\ &+\frac{\phi_2 l_2-\phi_1 l_1-\dot{x}^2}{V_x}\dot{\varepsilon}_d+\phi_1\delta\end{aligned} \quad (6)$$

$$\begin{aligned}\ddot{\varepsilon}_r = &-\frac{l_1 C_{\alpha f}-l_2 C_{\alpha r}}{I_z V_x}\dot{y}_v + \frac{C_{\alpha f} l_1 - l_2 C_{\alpha r}}{I_z V_x}\frac{(\dot{d}_v+\dot{x})}{V_x}\varepsilon_r \\ &-\frac{C_{\alpha f}(l_1^2-l_1 d_v)+C_{\alpha r}(l_2^2+l_2 d_v)}{I_z V_x}\dot{\varepsilon}_r \\ &-\frac{(C_{\alpha f} l_1^2+C_{\alpha r} l_2^2)}{I_z V_x}\dot{\varepsilon}_d+\frac{C_{\alpha f} l_1}{V_x}\delta\end{aligned} \quad (7)$$

where

$$\phi_1 = C_{\alpha f}\left(\frac{1}{m}+\frac{l_1 d_v}{I_z}\right), \quad \phi_2 = C_{\alpha r}\left(\frac{1}{m}-\frac{l_2 d_v}{I_z}\right)$$

Consider the effect of wind gust, etc, Eqs. (6) and (7) can be rewritten as

$$\begin{aligned}\ddot{y}_v = &-\frac{(\phi_1+\phi_2)}{V_x}\dot{y}_v + \frac{(\dot{d}_v+\dot{x}_c)}{V_x}(\phi_1+\phi_2)\varepsilon_r \\ &+\frac{\phi_1(d_v-l_1)+\phi_2(d_v+l_2)+2\dot{d}_v\dot{x}}{V_x}\dot{\varepsilon}_r \\ &+\frac{\phi_2 l_2-\phi_1 l_1-\dot{x}^2}{V_x}\dot{\varepsilon}_d+\phi_1\delta+d_1\end{aligned} \quad (8)$$

$$\begin{aligned}\ddot{\varepsilon}_r = &-\frac{l_1 C_{\alpha f}-l_2 C_{\alpha r}}{I_z V_x}\dot{y}_v + \frac{C_{\alpha f} l_1 - l_2 C_{\alpha r}}{I_z V_x}\frac{(\dot{d}_v+\dot{x}_c)}{V_x}\varepsilon_r \\ &-\frac{C_{\alpha f}(l_1^2-l_1 d_v)+C_{\alpha r}(l_2^2+l_2 d_v)}{I_z V_x}\dot{\varepsilon}_r \\ &-\frac{(C_{\alpha f} l_1^2+C_{\alpha r} l_2^2)}{I_z V_x}\dot{\varepsilon}_d+\frac{C_{\alpha f} l_1}{V_x}\delta+d_2\end{aligned} \quad (9)$$

Combine Eqs. (5), (6) and (9), we can obtain the following nonlinear state space equation

$$\dot{x}(t) = F(x(t)) + G(x(t))u(t) + Dw(t) \quad (10)$$

where the state vector $x(t)$, the vector field $F(x(t))$ and $G(x(t))$, the control command $u(t)$ and the uncertain disturbance $w(t)$ are defined, respectively.

IV. Controller Design

Our design objective is to specify the control command $u(t) = Kx(t)$ for nonlinear automated highway tracking system (10) via state feedback so that the controlled variable $x(t)$ can achieve the desired value. In this study, suppose x and y can be measured directly by GPS, and other states can be measured by measurement devices and corrupted by uncertain external noise, i.e.,

$$\begin{aligned}\dot{x}(t) &= F(x(t)) + G(x(t))u(t) + w(t) \\ y(t) &= Cx(t) + n(t)\end{aligned} \quad (11)$$

Where C is the identity matrix and $n(t)$ denotes the external noises in navigation process, whose statistical characteristics are not known with certainty. In general, it is not easy to treat the nonlinear optimal tracking problem of the automated highway system directly in (10) via output feedback under uncertain disturbance $w(t)$ and measurement noise $n(t)$. In this situation, the following Takagi-Sugeno fuzzy model is introduced to approximate the nonlinear automated highway tracking system (11) via interpolation.

Plant Rule i:

$$\begin{aligned}&\text{If } z_1(t) \text{ is } G_{i1} \text{ and } \dots \text{ and } z_g(t) \text{ is } G_{ig} \\ &\text{Then } \dot{x}(t) = A_i x(t) + B_i u(t) + Dw(t) \\ &\quad y(t) = Cx(t) + n(t), \ i = 1, \dots, l.\end{aligned} \quad (12)$$

where G_{ij} is the fuzzy set, $A_i \in R^{4\times4}$, $B_i \in R^{4\times1}$, l is the number of fuzzy rules, and $z_1(t), \dots, z_g(t)$ are the premise variables.The overall fuzzy system in (12) can be rewritten by

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{l} h_i(z(t))\{A_i x(t) + B_i u(t)\} \\ &\quad + Dw(t) \\ y(t) &= Cx(t) + n(t)\end{aligned} \quad (13)$$

where

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{l}\mu_i(z(t))}$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} G_{ij}(z_j(t))$$

$$z(t) = [z_1(t) \quad z_2(t) \quad \cdots \quad z_g(t)]$$

and $G_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in G_{ij}. It is natural to assume,

$$\mu_i(z(t)) \geq 0, i = 1, \dots, l$$

for all t. Therefore, we get the certainty functions as,

$$h_i(z(t)) \geq 0 \quad (14)$$

for $i = 1, 2, \dots, l$ and

$$\sum_{i=1}^{l} h_i(z(t)) = 1 \tag{15}$$

The state variable $x(t)$ is needed to be estimated for state feedback control. A fuzzy observer-based feedback control is introduced to solve this nonlinear noisy output feedback control problem.

Suppose the following fuzzy linear observer is proposed to deal with the state estimation of nonlinear automated highway tracking system (11), then,

Observer Rule i:

$$\begin{aligned}&\text{If } z_1(t) \text{ is } G_{i1} \text{ and } \ldots \text{and } z_g(t) \text{ is } G_{ig}\\ &\text{Then } \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\end{aligned} \tag{16}$$

where $\hat{y}(t) = C\hat{x}(t)$, L_i are the observer parameters (for $i = 1,\ldots l$).The overall fuzzy observer in (16) can be rewritten as follows

$$\dot{\hat{x}}(t) = \sum_{i=1}^{l} h_i(z(t))\{A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\} \tag{17}$$

Remark 1: For the convenience of design, the estimated state variables are specified as the premise variables, i.e., $z(t) = \hat{x}(t)$.

Suppose the following observer-based state feedback fuzzy controller is employed to deal with the above control system design, then,

Control Rule j:

$$\begin{aligned}&\text{If } z_1(t) \text{ is } G_{j1} \text{ and } \ldots \text{and } z_g(t) \text{ is } G_{jg}\\ &\text{Then } u(t) = K_j\hat{x}(t),\ j = 1,\ldots,l\end{aligned} \tag{18}$$

Hence, the overall fuzzy controller in (17) can be given by,

$$u(t) = \sum_{j=1}^{l} h_j(z(t))K_j\hat{x}(t) \tag{19}$$

where $h_j(z(t))$ is defined as in Eqs. (14) and (15) and K_j, $j = 1,\ldots,l$ are the control parameters.

Let us denote the estimation errors as,

$$e(t) = x(t) - \hat{x}(t) \tag{20}$$

By differentiating (21), we get

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t)\\ &= F(x(t)) + G(x(t))u(t)\\ &\quad -\sum_{i=1}^{l} h_i(z(t))\{A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\} + Dw(t)\\ &= \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\{(A_i - L_iC)e(t) + \Delta f - L_i n(t)\}\\ &\quad + Dw(t)\end{aligned} \tag{21}$$

where Δf and Δg denotes the fuzzy approximation errors as,

$$\Delta f = F(x(t)) - \sum_{i=1}^{l} h_i(z(t))A_i x(t)$$

and

$$\Delta g = G(x(t)) - \sum_{i=1}^{l} h_i(z(t))\sum_{j=1}^{l} h_j(z(t))B_iK_j x(t) \tag{22}$$

Suppose there exists a positive upper bound α so that,

$$\|\Delta f\| \le \alpha\|x(t)\| \text{ and } \|\Delta g\| \le \alpha\|x(t)\| \tag{23}$$

According to the inequality Eq. (23), we get,

$$\begin{aligned}(\Delta f)^T(\Delta f) &= \left\{F(x(t) - \sum_{i=1}^{l} h_i(z(t))A_i x(t))\right\}^T \cdot\\ &\quad \left\{F(x(t) - \sum_{i=1}^{l} h_i(z(t))A_i x(t))\right\} \quad \text{and}\\ &\le \alpha^2 x^T(t)x(t)\\ (\Delta g)^T(\Delta g) &\le \alpha^2 x^T(t)x(t)\end{aligned} \tag{24}$$

The augmented system is then equivalent to the following form:

$$\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}\sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\{A_i\hat{x}(t)\\ \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\{(A_i - L_iC)e(t)\end{bmatrix} \tag{25}$$

$$\begin{matrix}+B_iK_j\hat{x}(t) + L_iCe(t) + L_in(t)\}\\ +\Delta f - L_in(t)\} + Dw(t)\end{matrix}$$

After some manipulations, (25) can be expressed in the following form;

$$\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{e}(t)\end{bmatrix} = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\left\{\begin{bmatrix}(A_i + B_iK_j) & L_iC\\ 0 & (A_i - L_iC)\end{bmatrix}\cdot\begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}0\\ \Delta f\end{bmatrix} + \begin{bmatrix}0\\ \Delta g\end{bmatrix}\right\} + \begin{bmatrix}L_in(t)\\ -L_in(t) + Dw(t)\end{bmatrix} \tag{26}$$

Let us denote,

$$\tilde{x}(t) = \begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix},\ \bar{A}_{ij} = \begin{bmatrix}(A_i + B_iK_j) & L_iC\\ 0 & (A_i - L_iC)\end{bmatrix},$$

$$\Delta\tilde{f} = \begin{bmatrix}0\\ \Delta f\end{bmatrix},\quad \Delta\tilde{g} = \begin{bmatrix}0\\ \Delta g\end{bmatrix},\quad M_i = \begin{bmatrix}L_i & 0\\ -L_i & D\end{bmatrix},$$

$$\tilde{w}(t) = \begin{bmatrix}n(t)\\ w(t)\end{bmatrix},$$

$$\tilde{K}_j = \begin{bmatrix}K_j & 0\\ 0 & 0\end{bmatrix},\ \tilde{u}(t) = \begin{bmatrix}u(t)\\ 0\end{bmatrix}. \tag{27}$$

where

$$\begin{aligned}\Delta\tilde{f}^T\Delta\tilde{f} &= \Delta f^T\Delta f\\ &\le \alpha^2 x^T(t)x(t) = \{\alpha[\hat{x}(t) + e(t)]\}^T\cdot\\ &\quad \{\alpha[\hat{x}(t) + e(t)]\}\\ &= \alpha^2\tilde{x}^T(t)\tilde{x}(t)\end{aligned}$$

and

$$\Delta\tilde{f}^T\Delta\tilde{f} \le \alpha^2\tilde{x}^T(t)\tilde{x}(t) \tag{28}$$

Therefore, the augmented system defined in (26) can be expressed in the following form:

$$\begin{aligned}\dot{\tilde{x}}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\{\bar{A}_{ij}\tilde{x}(t) + \Delta\tilde{f}\\ + M_i\tilde{w}(t)\}\end{aligned} \tag{29}$$

Because the uncertain external disturbance $w(t)$ and measurement noise $n(t)$ are uncertain but bounded (thus the external disturbance $\tilde{w}(t)$ is also uncertain) and fuzzy approximation errors $\Delta\tilde{f}$ and $\Delta\tilde{g}$ are assumed to be bounded by (28). The H_∞ control has been shown to be an effective control scheme to attenuate the effect of uncertain $\tilde{w}(t)$, $\Delta\tilde{f}$ and $\Delta\tilde{g}$ to achieve a desired tracking performance. The following H_∞ tracking performance is considered as a design objective

$$\frac{\int_0^{t_f}\left[\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)+\tilde{u}^T(t)\tilde{R}\tilde{u}(t)\right]dt}{\int_0^{t_f}\tilde{w}^T(t)\tilde{w}(t)dt}\le\rho^2 \qquad (30)$$

$$,\forall\tilde{w}(t)\in L_2\left[0,t_f\right]$$

The H_∞ tracking performance in Eq. (31) can be rewritten as,

$$\int_0^{t_f}\left[\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)+\tilde{u}^T(t)\tilde{R}\tilde{u}(t)\right]dt$$
$$\le\rho^2\int_0^{t_f}\tilde{w}^T(t)\tilde{w}(t)dt,\forall\tilde{w}(t)\in L_2\left[0,t_f\right]$$

Note that if the initial condition is considered, the above H_∞ tracking performance should be modified as,

$$\int_0^{t_f}\left[\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)+\tilde{u}^T(t)\tilde{R}\tilde{u}(t)\right]dt$$
$$\le\tilde{x}^T(0)\tilde{P}\tilde{x}(0)+\rho^2\int_0^{t_f}\tilde{w}^T(t)\tilde{w}(t)dt,\forall\tilde{w}(t) \qquad (31)$$
$$\in L_2\left[0,t_f\right]$$

where $\tilde{P}$ is a positive definite weighting matrix.

The design purpose of this study is to specify a fuzzy observer-based control law in Eq. (19) to stabilize the nonlinear automated highway tracking system in Eq. (11) with the guaranteed H_∞ tracking performance (32).

Theorem 1: Suppose the fuzzy control law (19) is employed in the nonlinear system (29), and $\tilde{P}=\tilde{P}>0$ is the common solution of the following Riccati-like inequalities,

$$\bar{A}_{ij}^T\tilde{P}+\tilde{P}\bar{A}_{ij}+2\alpha^2I+\tilde{P}(I+\frac{1}{\rho^2}M_iM_i^T)\tilde{P}+\tilde{Q}+\tilde{K}_j^T\tilde{R}\tilde{K}_j<0,$$
$$\text{for } i,j=1,2,\ldots l \qquad (32)$$

where $\tilde{Q}=\tilde{Q}^T\ge0$ and $\tilde{R}=\tilde{R}^T>0$. Then the closed-loop nonlinear automated highway system is quadratically stable in the absence of external disturbances and noises and the H_∞ control performance of (31) is guaranteed for a prescribed ρ^2 in the presence of external disturbance. Moreover, if there is in the absence of external disturbances and noises then the error state will be ultimately driven to zero.

V. SIMULATION RESULTS

From the simulation results shown in Figs.2 and 3, it is easy to find the proposed tracking control exhibits an excellent tracking ability under the effects of the uncertain disturbances as wind gust, fraction of road and so on. Hence, we can conclude the proposed design algorithm possesses significant advantages for the development of the new-generation smart vehicles.

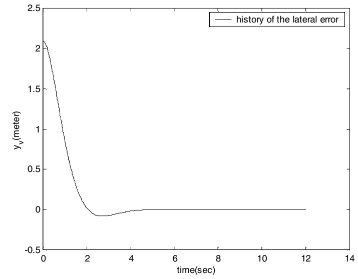

Figure 2. History of the lateral error

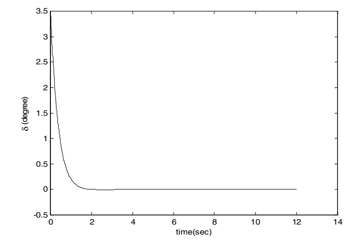

Figure 3. History of the steering angle

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