Analysis and Application of Image SVD as Filter Evaluation Standard

Chunping Wang, YuanZeng Cheng
Department of Optics and Electronics Engineering
Machine Engineering College
Shijiazhuang, 050000, China
E-mail: asajapa@sina.com.cn

Qiang Fu
Department of Computer Science and Technology
Tsinghua University
Beijing, 100084, China
E-mail: fu-q09@mails.tsinghua.edu.cn

Abstract—The effect of the image filter using SVD directly is strongly restricted by image interclass orientation and noise characteristic. By simulation and analysis, this paper present that not only the singular values but also the singular vectors are sensitive to above mentioned factors. And a method using the sensitivity is proposed to evaluate and select appropriate filtering methods quantitatively. The comparison is basing on the singular values of various filters outputs. The following simulation results show that the method can reflect the filtering effect of visual experience; it could be used for a large number of real-time image filtering and video filtering.

Keywords- image filter; SVD; Evaluation Standard

I. INTRODUCTION

The noise is unavoidable when the image signals is collected, converted and transmitted, especially when it is generated by photoelectric sensors. There is generally effective filter to reduce the noise before the follow-up transactions. The Singular Value Decomposition (SVD) is often used to reduce the dimensions of matrix and has become an extensive nonlinear image filtering method[1]. The Singular value and the singular vector correspond to different ingredient and characteristics of the image matrix. It is generally believed that the larger singular values and relevant singular vectors correspond to the real image signal while the smaller to the noise[2].

Therefore, according to a set of procedural rules, SVD filter set a threshold value and make the less singular values zero. Then it reconstructs the new singular value matrix and the original left and right singular vectors, thus the image denoising can be achieved. The basic mathematics model is summarized as follows[3]:

Suppose B comes from real image A with noise pollution:

$$B = A + N, B \in R^{I_1 \times I_2} \tag{1}$$

Where N is the random pollution. To improve filtering effectiveness, B is processed with SVD as:

$$B = U \cdot S \cdot V^T \tag{2}$$

$$U = [u_1, u_2 \ldots u_{I_1}] \in R^{I_1 \times I_1} \tag{3}$$

$$V = [v_1, v_2 \ldots v_{I_2}] \in R^{I_2 \times I_2} \tag{4}$$

$$S = \begin{bmatrix} S_1 & O \\ O & O \end{bmatrix} \tag{5}$$

U and V are called as the left singular vectors and the right singular vectors respectively. And both are orthogonal matrix. The S_1 is a sparse matrix as:

$$S_1 = \mathrm{diag}\,(\lambda_1, \lambda_2 \ldots, \lambda_R) \tag{6}$$

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_R > 0 \tag{7}$$

A suitable threshold ε is selected to separate larger singular values from others.

$$\lambda_i = \begin{pmatrix} \lambda_i, \lambda_i \geq \varepsilon \\ 0, \lambda_i \prec \varepsilon \end{pmatrix}, i = 1, 2 \ldots R \tag{8}$$

The new singular value matrix is :

$$\tilde{S} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots \lambda_r, 0, 0 \ldots), r \leq R \tag{9}$$

The corresponding new feature vectors can be combined into the result with less noise which can fast approximate to image signal:

$$\tilde{B} = U \cdot \tilde{S} \cdot V^T \tag{10}$$

II. SIMULATION ANALYSIS OF SVD FILTERING

This paper selects 50 typical sample images which are often used to test filters and then studies the laws of their singular values when they are mixed with different levels of Gaussian noise.

A. SVD Simulation

(a) Lena   (b) Football

Figure 1. The Original Images
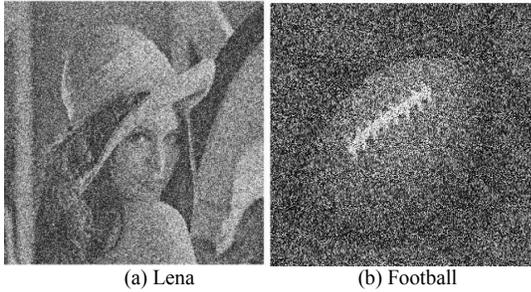
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(a) Lena   (b) Football

Figure 2. The Noise-polluted Images with $\sigma^2 = 0.3$

The Fig. 1(a) is the original *Lena* image which size is 480X480. (b) Football is 320X256. According to the basic model of SVD image filtering, it superimposed with Gaussian noise whose mean is 0 and variance σ^2 where $\sigma^2 = (0.01, 0.02 \ldots 0.3)$. SVD are implemented on the original image and the 30 results images separately.

Each image sequence includes 31 images, and 1550 SVD were conducted with above 50 samples.

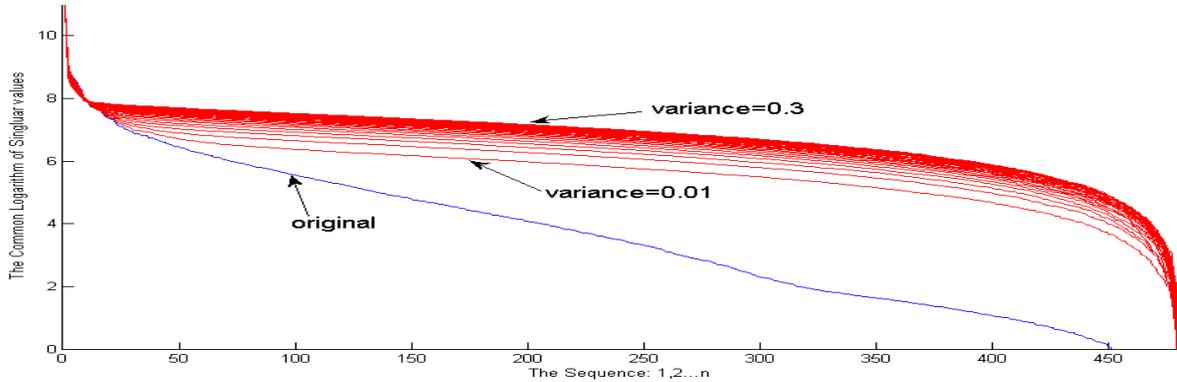Fig. 3 and Fig. 4 are the cross-sectional curves of singular values.

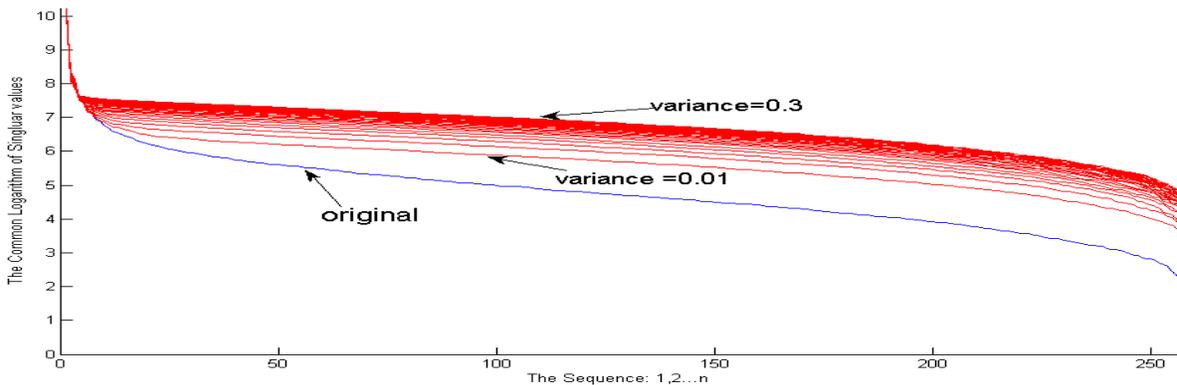Figure 3. The Singular Value Curve of Lena Sequence

Figure 4. The Singular Value Curve of Football Sequence

Simulation results show that:

- In a general way, the singular values of noise matrix are higher than of original matrix. The gap between two curves increases with decrease of the singular value. It means that the smaller singular values of real image, the bigger the above gap.
- For the superposition of noise with bigger variance, the curve is higher than that with smaller variance.
- For original image, even superposition of small noise, a drastic change will take place in its singular value curve. With the football image, Fig. 5 is the curves of singular values and Fig.6 shows the gaps generated by little noise ($\sigma^2 = 0.01$ and $\sigma^2 = 0.001$).
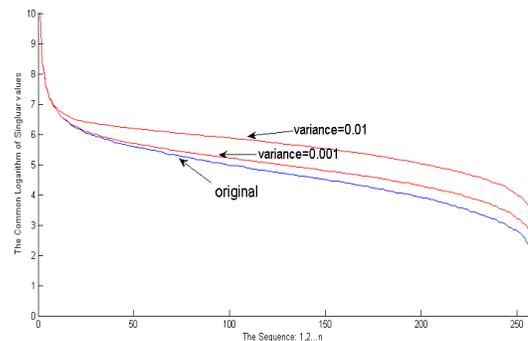

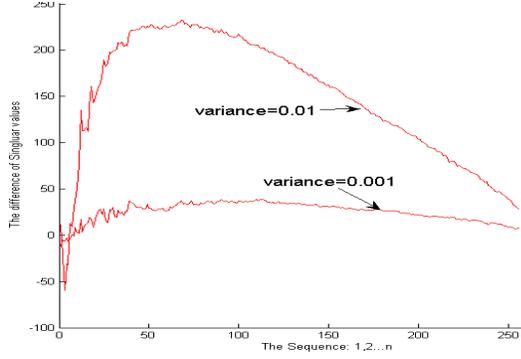Figure 5. The Singular Value Curve with Little Noise

Figure 6.   The Differeces of Singular Value Sequence with Little Noise

B.   Change of Singular Vectors with Noises

The basic idea of SVD filter is to set the smaller singular value as zero, and then reconstruct the image matrix with the light and the right singular vectors using formula (9). According to the above laws, the result is nearer to real image than the noise-polluted image.

It should be noticed that not only the singular values but also the singular vectors change with the noise, The U and V in the formula (9) may not equal to that in formula (2). The process of reconstruction needs to use real singular vectors with no noise.

For the following simulation, it assumes that the singular values are corrected well and equal to the original by the SVD filter $\hat{S}$. And reconstruction is as:

$$C = U \cdot \hat{S} \cdot V^{T} \qquad (11)$$

The result C just takes singular values into consideration. The comparison between C and A can indicate the change of singular vectors after noise pollution.

(1) Noise-polluted image   (2) Reconstructed image

Figure 7.   The Effect of Lena Reconstruction

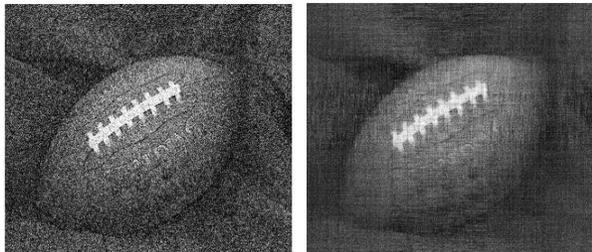(1) Noise-polluted image   (2) Reconstructed image

Figure 8.   The Effect of Football Reconstruction

Fig. 7 is the experiment result with noise variance 0.01 using formula (11). And Fig. 8 is with variance 0.05. The reconstructed images are distinct from their original even the singular values are quondam.

The comparison experiments show and confirm that[2]:

- The SVD is sensitive to the interclass correlation of images which means the similarity between nearby pixels. [3] pointed out that effect of SVD is restricted to the orientation of the image texture, and proposed an idea rotating the original images divided by corresponding blocks.
- The singular vectors changes with the noise, and it is an important factor to the filter effect.
- SVD filter which clears smaller singular values zero has contributed to improve the image quality, but the effect depends greatly on characters of noise and the image.

III.   SVD AS THE EVALUATION STANDARD

The conclusion comes out systematically by the analysis that the singular values of SVD are sensitive to the image texture. The differences between the singular values of the real image and these of noise-polluted are remarkable. Thus, the differences can represent the distance to the real one and the effect of image filter.

On the other hand, there are no uniform standards for effect of image filter. During image engineering practice, selecting filter often use the direct impression of experiments or similar experience. The conclusion confines to qualitative analysis, absence of persuasion and indication significance.

This paper presents a combination of other filters taking SVD as the evaluation standard. The following is the basic idea:

- Firstly, the potential filtering methods is selected, such as the neighborhood average, the mean filter, median filter.
- All above filter are implemented with the same noise-polluted image respectively. Assume the result sequence is {A_i} and there are i kinds of filters.
- SVD are applied to {A_i} respectively and generate the singular values sequence {S_i}.
- The best filter is corresponding to the singular values curve nearest to real curve by the comparison of {S_i}.

IV.   SIMULATION

A.   Simulation Model and Parameters

Below is the model to verify the algorithm with five kinds of average value of neighbor area filters [9].

$$f'(m, n) = \sum_{r=-k}^{k} \sum_{s=-l}^{l} f(m-r, n-l) \, H(r, s) \qquad (12)$$

The f(m,n) is the value of the m-th row and the n-th column pixel. The f'(m,n) is the output of the filter. H is

mask which illustrates the filter feature. Commonly used masks are:

$$H1=\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} H2=\frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} H3=\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$H4=\frac{1}{8}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} H5=\frac{1}{8}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{13}$$

The final goal is to compare effects of 5 filters with different masks.

B. Course and Result of Simulation

Fig 9 is the simulation result with the football image. The (a) is the original image then which is polluted by Gaussian noise, and the variance is 0.1. The (b) to (f) is the filter output of five masks.

(a) The Original Image (b) The Result of H1 Mask

(c) The Result of H2 Mask (d) The Result of H3 Mask

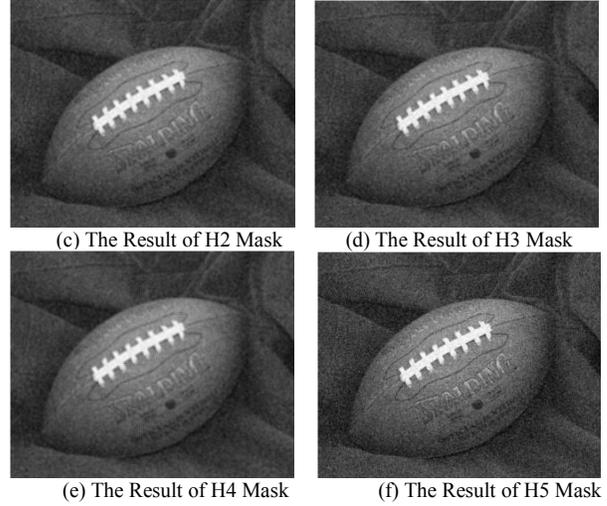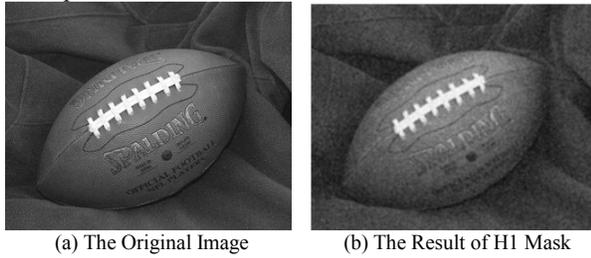(e) The Result of H4 Mask (f) The Result of H5 Mask

Figure 9.   The Effect of Football Reconstruction

Fig 10 is the singular value curves of above images. From the compression, all the rest curves are lower than the polluted. That means that all the filter-masts are effective and the noise is minimized in varying degrees. Further, the table 1 lists the distance from filter output curves to the original one. And the curve of H5 is nearest and should be the best filter to meet the sample image, and the conclusion agrees well with the visual effect.
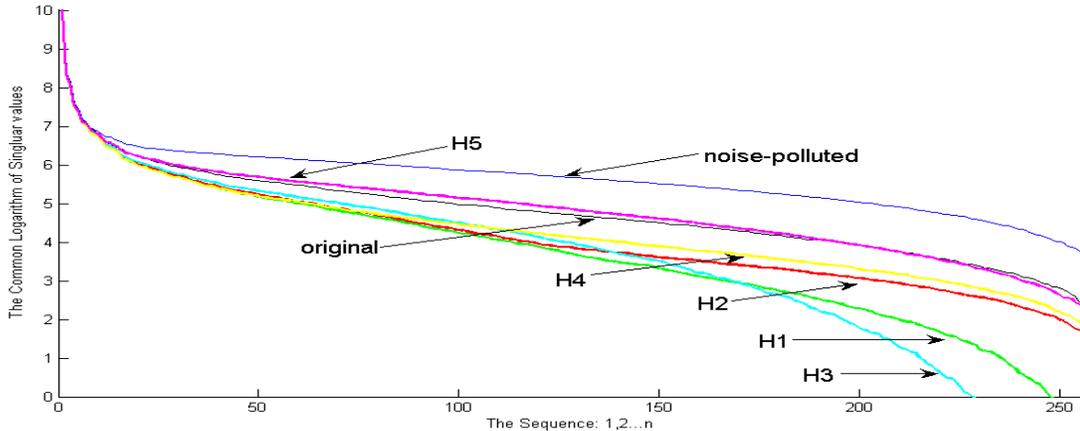

Figure 10. The Effect of Football Reconstruction

TABLE I.   THE SUM OF THE ABSOLUTE ERROR

Mask	H1	H2	H3	H4	H5
Error	15937	13736	13003	12822	4060

The simulation shows that SVD result could be used to measure the degree of filtering effect quantitatively.

V.   CONCLUSION

This paper analyzes the process and characteristics of SVD image filtering, points that SVD is sensitive to the correlation and texture of images and the state of noise. Based on this, an issue is proposed which take the singular values as the evaluation standard to compare image filters. This method can be applied to not only the filters of the same type, but also to the different kinds. Especially it is advantageous to select filter for video or other plenty of sequence images. And further study should be proposed on

its application in different cases, such as video and image sequence without real image as references.

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