# Optimization-Based Image Watermarking Scheme in the Wavelet Domain

Shuo-Tsung Chen
Department of Mathematics
Tunghai University
Taichung, Taiwan 407
shough33@yahoo.com.tw

Huang-Nan Huang
Department of Mathematics
Tunghai University
Taichung, Taiwan 407
nhuang@thu.edu.tw

Chih-Yu Hsu
Department of Information and Communication Engineering
Chaoyang University of Technology
Taichung County, Taiwan 413
tccnchsu@gmail.com

*Abstract*—This study presents an optimization-based image watermarking scheme by using Lagrange Principle. Unlike traditional way of single-coefficient quantization, this study uses multi-coefficients quantization to embed the watermarks. To modify the low-frequency coefficients of discrete wavelet transform (DWT), the Lagrange Principle is applied to derive an optimization-based formula. Experimental results show that the proposed method has good image quality under high embedding capacity and robust to JPEG compression.

***Keywords-Image watermarking; Lagrange Principle; DWT;***

## I. Introduction

With the development of internet, much digital information is widely used. For the digital information hiding and copyright protection of an image, digital watermarking method has been introduced. However, the hidden information or watermarking is easily intercepted by unknown persons on the internet. Thus, an image watermarking scheme should possess the following requirements: imperceptibility between the watermarked and original image, robustness against malicious attacks [1-7].

Generally, image quality and robustness are typically measured by peak signal-to-noise ratio (PSNR) and bit error ratio (BER). To optimize the tradeoff between PSNR and BER, this study presents an optimization-based quantization scheme which is different from single-coefficient quantization [4]. First, the PSNR in wavelet domain is rewritten as a performance index. An optimization-based equation is then proposed to connect this performance index and amplitude-quantization constraint. Second, the Lagrange Principle is used to derive the optimal solution. The optimal result is applied to embed the watermarks. Simulation results indicate that the proposed scheme has good image quality under high embedding capacity and robust to JPEG compression.

The rest of this paper is organized as follows. Section II rewrites PSNR as a performance index. An optimization-based equation that connects the performance index and amplitude-quantization constraint is proposed in section III. Besides, the Lagrange Principle is used to solve the optimization-based problem. Finally, the solution is applied to embed the watermark. In detection, watermark can be extracted without original image. Section IV does some experiments to test the performance of the proposed scheme. Conclusions are finally drawn in Section V.

## II. THE PROPOSED OPTIMIZATION-BASED PROBLEM

### *A. Discrete-time wavelet transform (DWT)*

The wavelet transform is obtained by a single prototype function $\psi(x)$ which is regulated with a scaling parameter and shift parameter [16]. The discrete normalized scaling and wavelet basis function are defined as

$$\varphi_{i,n}(t) = 2^{i/2} h_i(2^i t - n) \tag{1}$$

$$\psi_{i,n}(t) = 2^{i/2} g_i(2^i t - n) \tag{2}$$

where $i$ and $n$ are the dilation and translation parameters; $h_i$ and $g_i$ are low-pass and high-pass filters. In this paper, we use the wavelet bases in (1) and (2) to transform the host image into the orthogonal DWT domain by four-level decomposition. A method to implement DWT is a filter bank that provides perfect reconstruction. DWT has local analysis of frequency in space and time domain and it gets image multi-scale details step by step. If the scale becomes smaller, every part gets more accurate and ultimately all images details can be focalized accurately. If DWT is applied to an image, it will produce high-frequency parts, middle-frequency parts, and a lowest-frequency part. In order to guarantee the robustness, this study embeds the watermark into the coefficients in lowest-frequency part.

### *B. Optimization-based problem*

Generally, the quality of the watermarked image is evaluated subjectively by peak signal to noise ratio (PSNR) which is introduced as follows. If $I(i,j)$ and $\tilde{I}(i,j)$ are the

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values of the original and the modified pixel before and after watermarking, then PSNR is defined as

$$PSNR = 10\log_{10}\left(\frac{255^2 MN}{\left\|\tilde{I}(i,j)-I(i,j)\right\|_2^2}\right) = -10\log_{10}\left(\frac{\left\|\tilde{I}(i,j)-I(i,j)\right\|_2^2}{255^2 MN}\right) \qquad (3)$$

where $MN$ is the size of image, respectively. To achieve strong robustness, the lowest-frequency sub-band coefficients in DWT are used to embed the watermarks. After transforming the host image by DWT, we group every $k$ absolute values of lowest-frequency coefficient into a vector form

$$C_k = \left[|c_1|, |c_2|, \cdots, |c_k|\right]^T \qquad (4)$$

Since we implement the DWT with orthogonal wavelet bases, another consideration of PSNR is shown as

$$PSNR = -10\log_{10}\left(\frac{\left\|\tilde{C}_k - C_k\right\|_2^2}{255^2 MN}\right) \qquad (5)$$

where $\tilde{C}_k$ is the watermarked wavelet-coefficient vector corresponding to $C_k$. To have the best watermarked image quality, equation (5) is rewritten as

$$f(\tilde{C}_k) = \frac{(\tilde{C}_k - C_k)^T(\tilde{C}_k - C_k)}{255^2 MN} \qquad (6)$$

Based on the performance index in (6), the optimization-based problem is in the following form.

$$\textit{minimize} \quad f(\tilde{C}_k) = \frac{(\tilde{C}_k - C_k)^T(\tilde{C}_k - C_k)}{255^2 MN} \qquad (7a)$$

$$\textit{subject to} \quad g(\tilde{C}_k) = 0 \qquad (7b)$$

where the equation in (7b) is an embedding equation, respectively.

## III. EMBEDDING AND EXTRACTION PROCESS

Based on the optimization-based quantization problem in (7), this section introduces the proposed embedding and extraction process.

### *A. Embedding process*

Unlike the traditional single-coefficient quantization, this work uses amplitude quantization to embed each watermark bit. First at all, the watermark $\mathrm{B}=\{\beta_i\}$ is randomly generated by using a binary sequence. Hence the watermark values belong to the set {1,0} and is adopted as the secret key $K_1$. The proposed embedding technique is given as follows.

- If the embedded bit "$\beta_i = 1$" is embedded into $C_k$, the group-energy amplitude of the coefficients in $C_k$ is quantized to

$$GE1 = \left\lfloor \sum_{i=1}^{k}|c_i| \Big/ Q \right\rfloor Q + \frac{3}{4}Q \qquad (8)$$

- If the embedded bit "$\beta_i = 0$" is embedded into $C_k$, the group-energy amplitude of the coefficients in $C_k$ is quantized to

$$GE0 = \left\lfloor \sum_{i=1}^{k}|c_i| \Big/ Q \right\rfloor Q + \frac{1}{4}Q \qquad (9)$$

where $\lfloor\ \rfloor$ indicates the floor function, and $Q$ is the quantization size which is adopted as another secret key $K_2$. According to (8) and (9), the embedding technique can be rewritten as an equation $g(\tilde{C}_k) = 0$, that is

$$g(\tilde{C}_k) = L\tilde{C}_k - GE1 = 0, \text{ if "}\beta_i\text{=1".} \qquad (10)$$

or

$$g(\tilde{C}_k) = L\tilde{C}_k - GE0 = 0, \text{ if "}\beta_i\text{=0".} \qquad (11)$$

where $L_{1\times k} = [1.0 \;\; 1.0 \;\; \cdots \;\; 1.0]$ is the weighting matrix. Based on the performance index $f(\tilde{C}_k)$ in (6) and the constraint $g(\tilde{C}_k)$ in (10), the optimization-based quantization problem is in the following form.

$$\textit{minimize} \quad f(\tilde{C}_k) = \frac{(\tilde{C}_k - C_k)^T(\tilde{C}_k - C_k)}{255^2 MN} \qquad (12a)$$

$$\textit{subject to} \quad g(\tilde{C}_k) = L\tilde{C}_k - GE1 = 0 \qquad (12b)$$

To embed the watermark $\mathrm{B}$, we need to solve optimization problem (12). By using Lagrange Principle, the optimal solution of the modified coefficients $\tilde{C}_k^*$ is

$$\tilde{C}_k^* = C_k - L^T(LL^T)^{-1}(LC_k - GE1) \qquad (13)$$

The bit "$\beta_\mathrm{i}$=1" can be embedded by using this optimal modified coefficients $\tilde{C}_k^*$ respectively. In other words, we can embed the bit "$\beta_\mathrm{i}$=0" by using $GE0$ instead of $GE1$ as follows.

$$\tilde{C}_k^* = C_k - L^T(LL^T)^{-1}(LC_k - GE0) \qquad (14)$$

### *B. Extraction process*

To extract the watermarked bits, we group every $k$ consecutive lowest-frequency coefficients into a set:

$$\tilde{C}_k^* = \left\{|\tilde{c}_1^*|, |\tilde{c}_2^*|, \cdots, |\tilde{c}_i^*|, \cdots, |\tilde{c}_k^*|\right\}, \qquad (15)$$

where $\tilde{c}_i^*$ denote the watermarked coefficient which is optimized. Then, the binary bits are extracted by using the following rules.

- If

$$\sum_{i=1}^{k}\left|\tilde{c}_i^*\right| - \left\lfloor \frac{\sum_{i=1}^{k}\left|\tilde{c}_i^*\right|}{Q} \right\rfloor Q \ge \frac{Q}{2}, \qquad (16)$$

the extracted value $\hat{\beta}_i = 1$.

- If

$$\sum_{i=1}^{k}\left|\tilde{c}_i^*\right| - \left\lfloor \frac{\sum_{i=1}^{k}\left|\tilde{c}_i^*\right|}{Q} \right\rfloor Q < \frac{Q}{2}, \qquad (17)$$

the extracted value $\hat{\beta}_i = 0$.

Finally, the watermarked bits are all extracted as $\hat{B} = \left\{\hat{\beta}_i\right\}$.

## IV. EXPERIMENTAL RESULTS

To examine the performance, an image *Lena* of size 512×512 is adopted as the example image. The host image is decomposed into 4 levels by DWT transform and then the watermark is embedded into the lowest-frequency coefficients respectively. Fig. 1 and Fig. 2 show the *Lena* ‘s original image and the watermarked image with k=4, Q=60. The PSNR of the watermarked image is 44.9.

Fig. 1. Original image.

Fig. 2. Watermarked image.

The robustness is measured in term of bits-error-rate (BER) which is defined as

$$\text{BER} = (\text{Number of error bits}/\text{Total bits}) \times 100\%$$

JPEG compression is the most popular image processing in recent years. Hence we use this compression to test the robustness of the proposed method. The BERs under JPEG compression with quality factors 7, 10, 12 are 2.02, 0.26, 0.

## V. CONCLUSION

This study presents an optimization-based amplitude quantization technique for image watermarking. Based on an equation that connects watermarking cost function and amplitude quantization equation, we obtained an optimization-based formula for watermarking. The results show that the proposed method has high PSNR.

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